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JAN 30 1950

THE MATHEMATICS TEACHER

Volume XLIII JANUARY · 1950 Number 1

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OFFICIAL JOURNAL OF THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
525 WEST 120TH ST., NEW YORK 27, N.Y.

Printed at Menasha, Wisconsin

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

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Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to the office of

THE MATHEMATICS TEACHER
525 West 120th St., New York 27, N.Y. (Editorial Office)

Subscription to THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE $3.00 PER YEAR *(eight numbers)* to individuals;
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THE MATHEMATICS TEACHER

Volume XLIII Number 1

Edited by William David Reeve

What Kind of Geometry Shall We Teach?

By M. Van Waynen
Berkeley High School, Berkeley, Calif.

Several years ago I wrote an article[1] in which comparison was made between the merits of the traditional geometry course and the newly inspired course which focuses attention on the principles of logical thought and its application to everyday situations. Needless to say, my article favored the latter. Several experiments[2] have been conducted to substantiate this choice, each one proving to a statistician's satisfaction that the pupils of the experimental classes acquire the usually attained geometric achievement while definitely managing to surpass the control classes with scores of nearly two to one in reasoning achievement. In spite of the evidence at hand, however, what if anything is being done to improve the situation? Very little it seems.

Many teachers have said with ample justification that our educational system is at least fifty years behind the times. Yet, enlightened professionals that we are, we do so little about it. Should we continue teaching geometry that has so little value for the greater majority of our pupils, or shall we teach a course from which everyone can profit no matter what his goal in life? The stimulus of the war cannot always be relied upon to keep our geometry enrolment up. But of greater importance, the time has come to give of our bounty to all. Geometry is the ideal vehicle for teaching the simple pattern of clear thinking and its application to the vital problems of everyday life.

The contention of my first article was that transfer to other fields of thought can be made easily. Many are still not convinced. Perhaps another analysis of the thought processes used in geometry supplemented by actual examples of everyday application will help bring the disbelievers into the fold. Repetition is a powerful educational tool!

From the fact that the pupils who pass through our traditional geometry courses retain only the foggiest notions of the logical patterns which geometry so simply and obviously presents, one might deduce that the teachers of geometry are themselves not aware of it or find it too much trouble to present. The second alternative is inconceivable because our schools were created supposedly for the purpose of

[1] "How Shall Geometry Be Taught?", by M. Van Waynen, Mathematics Teacher, February 1944.

[2] "The Nature of Proof," by Harold P. Fawcett, The Thirteenth Yearbook, The National Council of Teachers of Mathematics.

"Teaching Geometry to Cultivate Reflective Thinking, An Experimental Study with 1239 High School Pupils," by Gilbert Ulmer, Journal of Experimental Education, 8: 18–25.

teaching our young citizens to think!

Figure I pictures once again the thought pattern presented by geometry. Note how out of our heritage of experience are born the undefinable concepts so glibly bandied about in an absolute sense. On these are built our definitions. Relationships are then noted by the observant person who later develops certain suppositions or assumptions. If interested enough that person could go a step further and attempt to prove his theories. He might do this as the scientist generally does through the experimental or inductive method or as the mathematician does, through the inductive to the deductive method. Here he avails himself of the indirect or direct methods, the direct method admitting of two paths, the synthetic or analytic. The whole chain so set up is open to attack at several vulnerable spots, especially in the undefinables and assumptions since they depend so much on our background and experience, where one can be so easily led astray. Hanging our whole framework of geometric reasoning onto the nebulous clouds of man's experience should serve to remind all that the absolute verity of our conclusions may always be questioned but that the reasoning applied in deriving them can be quite sound.

Little reference will be made in this paper regarding undefinables and definitions. However, it must again be emphasized that non-science teachers are not all aware of the meaning of definition as used by the scientist and mathematician. A

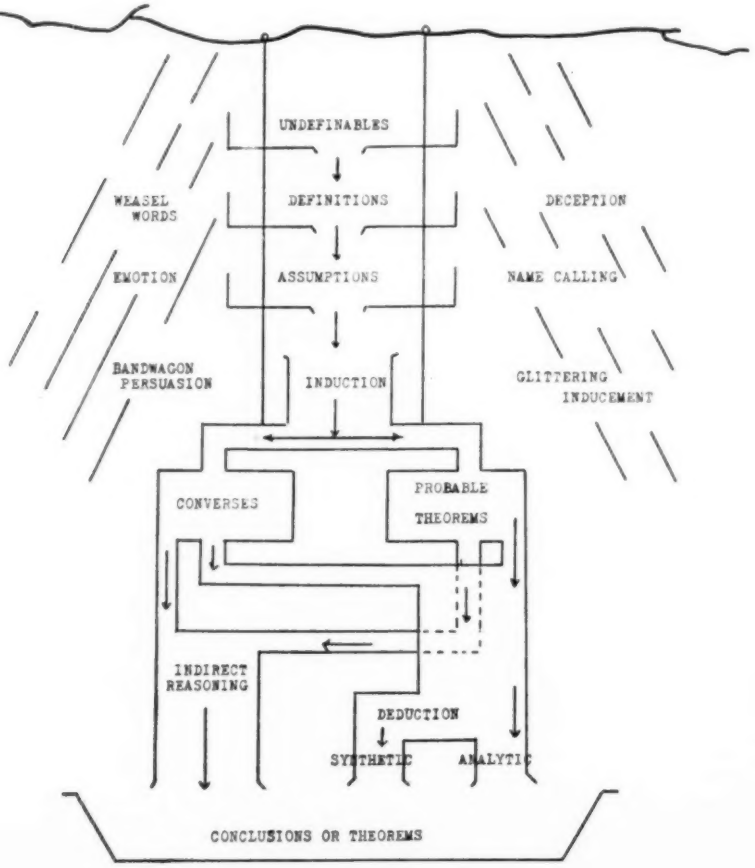

PLATE I. Experience of man.

pupil once defined 'energy' as 'the power to do work'. His physics teacher promptly marked him wrong; his English teacher, to whom the pupil appealed the following day, assured him the statement was correct. Some geometry teacher had not made the proper impression in between. Some of you will remember the story of the young western lawyer who fell in with a group of big city "intellectuals" who were talking fast and furious about Russia, "the proletariat," "the bourgeoisie" and "the Communist Party Line." The lawyer listened attentively. Then he drawled: "Gentlemen, this is a new kind of language. Out where I come from the "party line" means a rural telephone line with ten to twenty subscribers." Yes, geometry can be fun!

There is an excellent way by which teachers can make their pupils superconscious of hidden assumptions. Many an argument can be settled by noting how absurd and fallacious underlying assumptions can be. The device that can be used here to the great delight of one's class is the study of propaganda. Such a study is not as out of place here as some might think. For an outline one can use the pamphlet published by The Institute for Propaganda Analysis, Monthly Letter, Vol. I, No. 2, a short treatise giving the definitions and illustrations for seven easily recognized devices of propaganda, namely:

1. Name Calling
2. Plain Folks
3. Glittering Generalities
4. Transfer
5. Card Stacking
6. Band Wagon
7. Testimonial

Examples of simple propaganda can most readily be found in our magazine advertisements. Our advertising agencies are very aware of all the tricks of propaganda and apparently consider their use legitimate. Certainly the teaching profession should not fear debunking a few tobacco and liquor ads. A great deal of satisfaction can be derived from trying to get pupils to justify the smoking of a cigaret when such an illustrious hero as Hank Greenberg can be enmeshed in such folly as follows:

Greenberg Strikes Out

On page 23 of the April 15 issue of the Detroit *News* was this ad:

HANK GREENBERG GOES ALL OUT FOR RALEIGHS

"There's no hocus-pocus about it!" says Hank Greenberg, baseball's home-run star. "I've read the reports and Medical Science has proved you can't beat Raleighs for *less nicotine . . . less throat irritants . . .* all-round *safer smoking*! I recommend Raleighs to all my friends. Raleighs are right!"

On page 17 of the same issue is this interview with Hank Greenberg:

"I feel fine now—better than I have for some time," said Greenberg. "The doctors said I had a stomach disorder and I'm giving up cigarettes and coffee. I never was much of a smoker anyway. I'm sleeping better now and I feel much better."

—Contributed by Mark Brandewiede[3]

Obviously card stacking devices such as these serve to point out that we cannot accept the assumption that our athletes, movie stars, crooners, etc. are reliable people. True, the assumption made here that we cannot rely on what is said in ads is a generalization derived on inductive reasoning. But the examples of this type of propaganda are so numerous that the assumption made can be considered to have a high degree of validity. Perhaps less evil is done when recommendations are made for breakfast foods, etc. but one could say the same thing for tobacco and liquor. There are those who might recommend narcotics.

In the criticism to a beer ad one pupil wrote:

In this beer advertisement quite a fuss is made over America, and what a fine place it is, but not much is said about the beer that they are trying to sell. It speaks of our "fore fathers" and "natural resources," of "comforts and conveniences," and of "the spirit of American pioneers," but what has all of this to do with the sale of beer? Such things are put in advertisements not to make the reader go right out to buy the product, but to put him in a good frame of mind for the sales talk. What they say about America is all very true, and most of the

[3] *Readers Digest*, July 1946: 56.

people that read this advertisement will agree with what it says. After reading the first part of the advertisement, without thinking, the reader will assume the advertisement's truthfulness in the first part will be carried over to the remaining part. This may or may not be the case.

At the bottom of the page a statement is made that is supposed to convince you that their beer is the best beer that you can buy. They say, "Drink . . . for five days, and on the sixth day try to drink sweet beer. It assumes that it is the only beer that is not sweet. This statement does not take into consideration that if a person is drinking a sweet beer it will be just as hard to change to . . . as it would be to change from . . . to a sweet beer. The advertisement does not, therefore, expect the reader to make the test.

This, as beer advertisements go, is a fairly good one, for it does not make any absolutely untrue statements, as is done in some of the other beer advertisements. It does not, however, give any reasons why you should buy their beer, and it does not attempt to show why . . . is better than other beers. Almost all advertisements of this type make very general statements with a lot of omitted assumptions and flowery sentences. They give no facts. They rely on the prospective buyer to read the ad without thinking much about it and taking all he reads for granted.

One student brought in two ads in which a rather famous aviator had endorsed two brands of cigarettes. This aviator had the good sense to put them in separate magazines. This subject could be pursued further with great gusto, but the following bit of humor ought to cinch the argument.

"To what do you attribute your long life?" the newspaper reporter asked the centenarian.

"I don't rightly know yet," replied the old-timer puffing lazily on his pipe. "I'm still dickering with two breakfast-food companies."

Capper's Weekly[4]

Geometry teachers have spent too little time on inductive reasoning. This seems unfair since all geometric theorems were derived originally through the experimental method. It would also assist the science teachers immensely if a little preparation in this type of thinking could be given in the mathematics classes. Besides this, and of the most importance to the mathematics teacher, is that a sound knowledge of inductive reasoning and its weaknesses builds a greater appreciation for the deductive.

One of the best examples of this is to be found in one of Al Capp's cartoons. This cartoon shows the secret headquarters of the spy system of a foreign power sending out two spies to get detailed information about the life and habits of the American people. Their report will determine, whether or not this foreign power will declare war on the U.S. or maintain peace. In the lair of the Yokums they encounter old ladies with Herculean strength capable of most any feat and fabulous gorilla-like characters such as Earthquake McGoon. One of the operatives manages to return with his report, although very much the worse for wear. On his one visit to what he thinks is a typical American community he concludes that an encounter with an American army composed of American old ladies and Earthquake McGoons would lead to national suicide for his country. This type of material is very popular with most young people taking geometry.

In the November 2, 1947 issue of Time in the section on Education we find:

Quis Custodiet . . . ?

It was the teachers' turn to be tested. In Denver, the Rocky Mountain News asked about 100 Colorado public elementary and high-school teachers such questions as who killed Lincoln? What was the Monroe Doctrine? The teachers average grade: 67 (below passing). Some believed that Aaron Burr assassinated Lincoln, that George Washington wrote the Declaration of Independence, and that the Civil War lasted ten years. Toughest question: Who Married Pocahontas? Most teachers said John Smith. Correct answer: John Rolfe.[5]

In the December 8, 1947 issue we find:

De Custodibus

Sir:

I wish to give you the facts in regard to a story which you carried in Nov. 3 issue of Time, under the heading "Quis Custodiet?"

The story, based upon an Associated Press release from Denver, stated that 100 Colorado public elementary and high-school teachers had been given a history test in Denver by the Rocky Mountain News, and that the average grade was 67.

The facts are that four teachers were given the test . . . . One of the four made a score of 20, the average for the four was a grade of 67, which would mean that the other three made an average grade of 83. So you see 9000 Colorado

[4] Quoted in "Life in These U.S." by Mrs. Edwin R. Ewers, *Readers Digest* O.'46: 108.

[5] Courtesy of Time, Copyright Time, Inc., 1947.

teachers were being judged as to their knowledge of history on the basis of a low score by one teacher.

Since the erroneous story as released by the Associated Press was carried in your columns, it seems only fair to Colorado teachers and teachers in general that you carry this correction as released by us.

Ward B. Kimball
Director of Publications and Publicity
Colorado Education Assn.
Denver.[6]

Is any comment necessary? Examples of this kind abound because this is how we generally derive our conclusions. Let us do something about it. Again let us indulge in the facetious.

If a man makes a stupid mistake, the other men say: "What a fool that man is." If a woman makes a stupid mistake, the men say: "What fools women are!"—Quoted by H. C. L. Jackson in Detroit *News*.[7]

There are many of us who are genuinely disturbed over the fact that ultra-modern music (not primitive jazz) rather grates on our ears. Modern composers seem to think that their music is quite listenable and that it can be enjoyed. They tell us that every composer of note was considered a heretic in his day, and that no progress in art has ever been made without overcoming extreme opposition, implying that every composer who meets with opposition is necessarily great.

Now we know that every modern composer will not be judged great; but some may. Beethoven, Wagner, Debussy, and others who were once considered arch-heretics, certainly proved their greatness. It is still possible that the seemingly heretical theories of Schoenberg and others, will be similarly accepted by all someday in the future. We do not want to appear unappreciative; we are left badly confused. After all there may be a limit beyond which musical distortions cannot go without protest from the ear.

The foregoing shows how easily the converses of generally accepted statements can be clothed with authority. Some even fail to recognize the fact that the theorem in this case is based on rather weak induction. Were all great composers martyrs?

Examples of indirect reasoning are common, perhaps too common, to be bothered with here. Did you by any chance read the short note on Russia in Time July 29, 1946 quoted below.

On to Odessa

Last week the Moscow censor passed a report that Marshal Georgi Zhukov, head of all Soviet land forces, had been assigned to command the Odessa military district. It was as though Ike Eisenhower had been sent back to San Antonio. The Soviet press did not mention the move, but the world outside erupted with rumors:

Was it part of Russia's war of nerves on Turkey? In 1940 Zhukov led the troops that took over Bessarabia and northern Bukovina from Rumania. Odessa is the military district nearest the Dardanelles.

Was the Communist Party "disciplining" the Red Army—the one power in Russia that might conceivably replace it? That seemed unlikely. Last week Red Star and Red Fleet, official organs of the Soviet Army and Navy, began a campaign to strengthen the Communist Party organization in the armed forces. Red Star called for improvement in the Army's "weak and timid" propaganda discussions, and said that to "guarantee our state interests" the Party must reach thousands of officers who lack "serious political education."

Did Stalin, like all dictators, believe in periodic "changing of the guard?" One Russian smiled, shrugged, quoted the aria in Tchaikovsky's Queen of Spades: "Today it is I, tomorrow, you . . . ."

Had Zhukov, whose broad bosom jangles from throat to waist with Soviet and Allied medals, been too friendly with the U.S. and British generals while he was in Berlin?

Whatever the reason for the shift, it seemed a good guess that with the war over, Russia wants her military leaders to take more and more of a back seat and mingle less and less with outsiders. From Washington last week came a significant story. When General Walter Bedell Smith, who had established close cordial contacts with many a Soviet brasshat in Berlin, reached Moscow as U.S. ambassador last Spring, he invited seven of them (including Zhukov) to dinner at the Embassy. Only one (not Zhukov) came.[8]

As teachers of Mathematics we do not go into the political aspects of such articles as quoted above, but we can recognize the reasoning for what it is and pass judge-

[6] Courtesy of Mr. Kimball and Time, Copyright Time, Inc., 1947.

[7] *Readers Digest*, Au.'46: 26.

[8] Courtesy of Time, Copyright Time, Inc., 1946.

ment on its potency. Did you read the article from the London Economist quoted in Time July 15, 1946 on "The Carrot and the Stick" based on the premise that economic activity is goaded on either by "incentive" or by the "stick" of economic or dictatorial compulsions? It is worth reading even though fallacies will be found in this example of indirect reasoning.

The piece de resistance of our geometry, even if the most difficult, is deduction. Here our path is strewn with endless frustration. But need it be as formidable as it seems? First examine Figures II and III to see how the skeleton of Figure I is given flesh and blood in geometry and in economics. Perhaps geometry will seem the easier of the two to you because of your experience with it; the other is just as easy if teaching for transfer is made a habit. The first few tries are the hardest! Please recall your consternation when you could not at first follow the new presentation of geometry made by Birkoff and Beatley in their new Basic Geometry. It was not so easy at first to assimilate because we had gotten ourselves into a rut.

Excellent examples of deduction on the High school level can be found here and there in our literature. "The Broken Window Fallacy" condensed from "Economics in One Lesson" by Henry Hazlitt in the Readers Digest, July 1946 is interresting as well as instructive and as deductive as it can be. The economic implication of a broken window is expertly and clearly traced from beginning to end. You will not only be teaching some geometry, but you will be teaching some much needed economics as well. One should not neglect the moral side of the question either. If our young people were made to

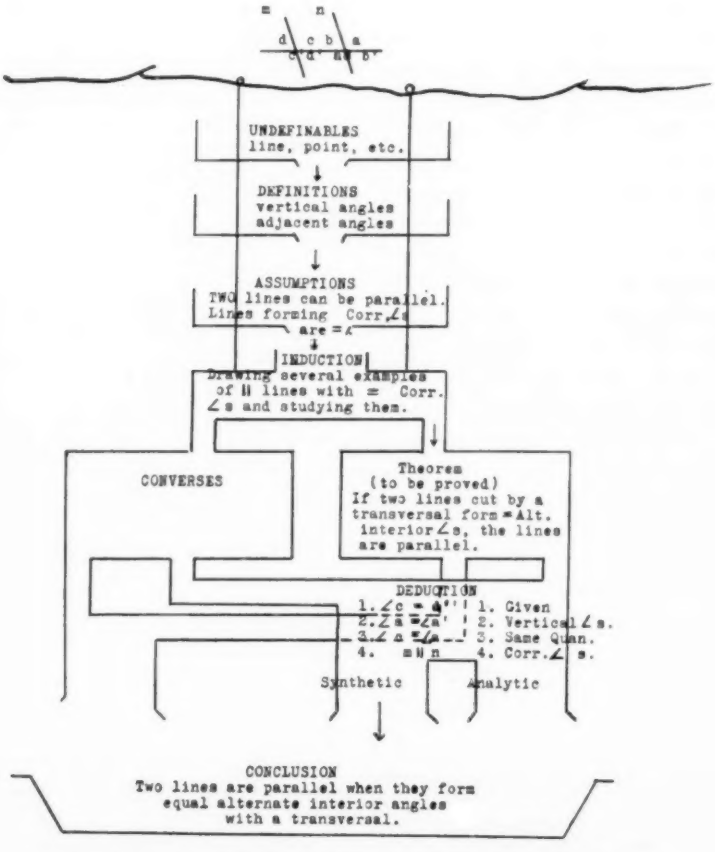

PLATE II. Geometry—Experience and background.

stop and consider the consequences of one foolhardy act, they might not be so prone to commit them.

The following is a more simple example. Can you see the deductive pattern set by Dean Acheson in the following? Could your pupils?

Breaking the Circle

Congress was increasingly critical of foreign loans made without advance assurances; the politico-economic situation in Greece was rapidly getting worse. Caught as usual between criticism and circumstances, the State Department last week announced that the Export-Import Bank had lent Greece $25 million. At the same time, the State Department told the Greek Government to pull up its socks; the loan would do no good if Greece did not take vigorous steps to revive industry, cut expenses and control inflation.

Asked how he would justify the loan to Congress, Acting Secretary of State Dean Acheson patiently explained about the cart and the horse: Greece could not stabilize unless she got the money, and reform without stability was impossible. The U.S. hope: with the Greek economy improved, elections could be successfully held, which in turn should ease the tense Greek political situation and contribute to recovery.[9]

When reading articles purporting to analyze complex situations, one is amazed at the amount of application one versed in the simple logic of geometry will automatically make. As an example of such application let us briefly review the article "World Gamble" contained in the Business section of Time, January 12, 1948.

This article can be considered to be a sane, critical analysis of 1947 economic conditions in the U.S., since those who wrote it are men highly trained and experienced in their field. One can read it expecting a fairly unbiased presentation based on established economic theory.

[9] Courtesy of Time, Copyright Time, Inc., Jan. 21, 1946, p. 24.

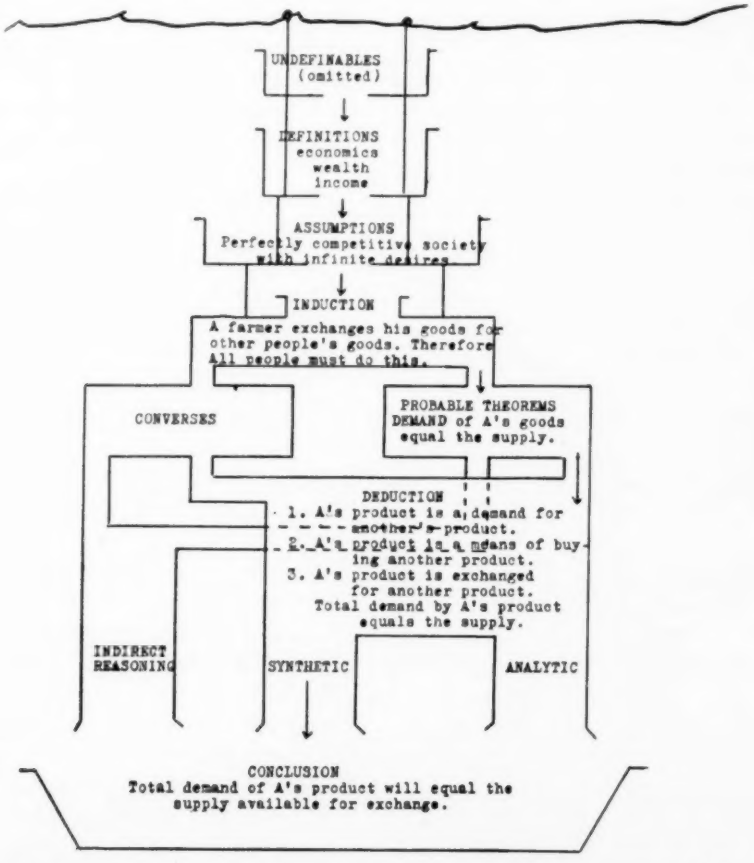

PLATE III. Economics background. (First chapter of text devoted to this.)

Too, no topic could be of more importance to the well being of our nation than that dealing with its economic problems. A good many of us must have read this article or articles like it.

Fortunately the authors of "World Gamble" point out or imply their facts and assumptions rather clearly. This is normally not the case. Readers usually have to put in a good deal of effort to find such information. The article purports to show that inflation would have been on its way out in 1947 if it had not been for the fallacious assumption that we could cure our economic ills by isolating our inflation from that of the world. The Marshall Plan brought forcibly to mind that this assumption could not be accepted. Secretary of State George Marshall was in effect the cause of the change in business thinking which kept prices from coming down.

The article transposed to the synthetic theorem presentation of our geometry texts might be written as shown below. An extra column has been added to contain supporting evidence quoted from the article itself[10] and comments that might be made by the reader of the article.

Theorems:[8]

(1) Because all controls were thrown off U.S. industry, the inflation existing at the beginning of 1947 was stopped by April 1947.

(2) Because the demand of foreign nations continued at a high peak, inflation still existed at the end of 1947.

Given:

(1) The cost of living went up from 153.3 to 166 during 1947 (1941 figure: 105.2).

(2) In throwing off all controls the U.S. had bet that industry could pour out enough good to lick the war time inflation.

(3) In terms of items turned out 1947's industrial production was 23% above 1946's surprising total.

(4) In April came the first wind of what seemed to be the recession storm. . . . U.S. production slid from the postwar high of 190 to 87. Industrial employment went down. Commodity prices, which had reached a postwar peak of 149.5 began to slide. . . . Soft goods sales slumped. . . . The big four (. . . .) cut tire prices. Cut also were prices of soap, radio sets, furniture, paints.

(5) At summer's end business began to build up their inventories. The drop in prices, such as it was, had not lasted long or gone far.

(6) The U.S. alone turned out well over 50% of the known industrial production of the world compared with 30% before the war.

(7) The (Marshall) Plan meant that the export boom was not going to collapse: foreign nations were going to get the cash to keep the boom going.

(8) For the 15 billion in U.S. goods almost one eleventh of the U.S. total sales—the world exported to the U.S. only 5.7 billion. . . .

(9) At summer's end, business men began to build up their inventories. The drop in prices had not lasted long or gone far. . . . So the U.S. paid for its first try at bailing out the world with higher prices all around.

(10) They helped pull up the wholesale commodity price index (1926 average: 100) from 141.5 to 163 in a year. . . . So the U.S. had its first $2.95 a bushel cash corn in its history; its first $3.00 wheat in 27 years.

The authors of this article go on to predict that E.P.R. would not increase overall U.S. exports even though the U.S. was handing out the cash to other nations to pay for the exports—thus would not increase inflation. A critical examination of the facts presented and the manner in which they are linked together to support this prediction provides another interesting exercise.

[10] Courtesy of Time, Copyright Time, Inc., 1948

Comprehensive work in all phases of logical thinking is provided through such tests as the "Interpretation of Data" tests printed by the Progressive Education Association. Their tests in the "Application of Certain Principles of Logical Thinking" are also excellent. If time would permit, some pupil work along this line might well be shown here. These exercises are not only highly instructive but also very practical and interesting—much more than geometry can be to most pupils.

Steps	Proof	Evidence and Comments
1. There was inflation at beginning of 1947.	(Def.) Inflation is an undue rise in the cost of living.	Given: (1) Article states: "Inflation is an excess of demand over supply. This is an assumption; not a definition. Perhaps this is so elementary we should overlook it.
2. Controls on industry were thrown off.	Given (2).	Were all controls thrown off? Farm subsidies could not be thrown off for several years. Probably some monopolistic controls continued to exist unchecked.
3. U. S. production rose.	An assumption. If an industry fettered with governmental controls is freed of such controls, production will rise.	Implicit in: Given (2). Backed up by: Given (3).
4. Prices of goods rose.	Given (1).	
5. The inflation was cured (by April).	An assumption. If the supply is made to equal or exceed the demand and if prices rise, inflation will cease.	Assumption implicit in: Inflation is an excess of demand over supply and one way in which the two can be brought into balance is by such a rise in prices that the available supply absorbs the demand. . . . Also to: Part of the drop was due to the increase in production which tended to satisfy demand and part to the rise in prices. Backed up by: Given (4).
1. However, at the end of 1947 the conclusion so aptly proved above was no longer true. Why? The chain was weak. Where? The supply had not met the demand or the price rise had not taken effect.	Given (5).	
2. The rest of the world did not produce its share of goods in the world's economy.	Given (6) and (8).	
3. Money was or seemed available to buy goods.	Given (7).	
4. European production remained below normal.	Given (8).	
5. The demand for goods from foreign countries remained high or even increased.	An assumption: If a nation does not produce up to its normal capacity and has ample funds provided, it will buy what it needs by buying from a foreign power.	Evidence, of course, is lacking due to the fact that the Marshall Plan had not actually been enacted. However, it did affect U. S. merchants and manufacturers profoundly. Given (9).
6. Inflation continued.	Given (10).	

Meaningful Division*

By Rolland R. Smith
Coordinator of Mathematics, Springfield, Mass.

Readiness for number concepts and number relationships is not so much a matter of chronological age as it is of experience. Of course, the pupil of the fourth or fifth grade can be taught division when it would be impossible to do this with the average first grade pupil. But it is not his age that makes this possible so much as it is the fact that having lived longer he has had time to gain greater experience with numbers. If we could blot out all the number experiences of a ten or eleven year old child, the chances are that he would not be more ready for division than his younger brother.

We must think in terms of providing experience. The kindergartern and the first grade teachers have to develop many ideas before they can present the fact that 3 plus 4 is 7. They have to look ahead three or four months, often longer, to the end goal they have in mind. You should never lose sight of the general principle involved in this situation. For example, instead of saying to yourself in the fourth grade, "Today I shall show my pupils how to do division," ask rather, "What experiences must I provide for my pupils before they can understand what I shall show them about division."

An older psychology warned us to teach every process in precisely the way it will be used. This psychology led to many of the difficulties we have in teaching arithmetic today. Refined methods were taught from the beginning. Pupils were told, "This is the way to do it," and then were drilled on the process until they mastered it *temporarily*. This kind of teaching required no great show of intelligence or initiative on the part of the teacher. Elementary knowledge of the subject matter and patience in drudgery sufficed. From this kind of teaching came the all too prevalent thought, "Anybody can teach arithmetic." A psychology that is much more acceptable to discerning teachers tells us that we must look ahead to what we desire our pupils to know and then provide carefully graded experiences leading the pupils from the point where they are to the point where we wish them to be. When the teaching of arithmetic is looked upon from this point of view, we see why the leaders in mathematical education insist that good work cannot be done unless teachers are willing to give the process of development the keenest kind of analysis.

The preceding paragraphs are a preamble to a discussion of the development of division with children in the first five grades. The teaching of division is often mechanical without much meaning behind it. An attempt has been made here to give the pupils a picture of division as a general concept and not as a series of steps to be memorized and to be changed in this detail and that for various types of division. It is always better to give pupils a large idea into which they can fit varying details than it is to give separate examples calling for mastery of the varying details in isolation. Before discussing this development, however, consider for just a minute the essential nature of division.

A Discussion of Division

(For the teacher, not the pupils)

Division is a series of subtractions just as a multiplication is a series of additions. In multiplication we have equal addends; in division we have, shall we say, equal *subtractees*. 17 divided by 3 is 5 with a remainder 2. As subtraction, it looks like the example on the next page. It is important to know that subtracting 3 five

* Read at the Annual Meeting of the National Council of Teachers of Mathematics at Baltimore, Md., April 1, 1949.

times in succession gives the same remainder as subtracting 5×3 all at once (Also shown at the right.)

These ideas are important in the understanding of division. Suppose I wish to divide 4926 by 24. I can easily see that 24 can be subtracted from 4926 one hundred times. I can do it by subtracting 24 one hundred times in succession or I can subtract 100×24. (Note that there is no problem as to where the 1 of the 100 should go in the quotient. It is 1 in hundreds' place.) I have a remainder of 2526 and that is the number with which I am now concerned. (See the examples below.) I see that 24 can be subtracted 100 times more. I subtract 100×24 again and the remainder is 126. I now see that 24 will not go 10 times. The remainder is small enough so that, if I wished, I could take out one 24 at a time. Instead of that, being a mature person, I shall subtract it 5 times at once; that is, I shall subtract 5×24. We have now subtracted our divisor 100 plus 100 plus 5 or 205 times and the remainder is 6. (Note that the zero in the quotient gives no difficulty.)

17	17
3	15
14	2
3	
11	
3	
8	
3	
5	
3	
2	

1.		2.		3.	
	100		100		205
24)4926			100		5
	2400	24)4926			100
	2526		2400	24)4926	
			2526		2400
			2400		2526
			126		2400
					126
					120
					6

This is the essence of division. Carefully developed, these ideas can be understood by elementary school pupils. The rest of this paper deals with the necessary development.

First and Second Grades

In the first and second grades "you should utilize the children's present interest to lay the foundation for meanings and skills that are essential to meet later needs."[1] The teacher should be able and always alert to recognize the arithmetic problems inherent in pupils' experiences. She can help pupils solve their immediate problems by using objects, pictures, diagrams, or other meaningful symbols to represent the number aspects of the situations giving rise to the problems. She does more, however, than solve the immediate problem. She uses the problem to help pupils understand the essential ideas in it, to let them hear the language that should be associated with it, and, by bringing in other similar problems, she generalizes the arithmetic relationships.

Of course, you will not teach formal division in the first or the second grade. But it is quite likely that the class may run into a problem where they would like to know how many 3's there are in 17. "How many tables are needed for 17 children if 3 are to sit at each table," could be such a problem. What should you as a teacher do in such a situation? "You can use the situation to help pupils understand the *meaning* of division."[2]

The number of 3's in 17 can be found by using pictures as shown below.

0 0 0/0 0 0/0 0 0/0 0 0/0 0 0/0 0

Pupils will see that there are five 3's in 17 and that 2 are left over.

Pupils can count backwards from 17, writing the numbers, and mark off the threes. 17 16 15/14 13 12/11 10 9/8 7 6/5 4 3/2 1. Again they will see there are five 3's in 17 and that 2 are left over.

Pupils can count back from 17 on a number chart and at the end of every three numbers make a mark. There will be five marks (/////) and there will still be two numbers to go. Note that the marking off

[1] Clark, Otis Hatton. Primary Arithmetic Through Experience, World Book Co., 1939, page 7.

[2] *Ibid.*, page 17.

of the circles and also the counting backward with the marking off of threes is a preliminary picture procedure leading to subtraction.

In teaching for meanings, you do not stop with the one example, 3's in 17. Pupils should learn that they have a method for any similar example, 5's in 21, 6's in 19, or 4's in 22. The important consideration at first is, not that the pupils should remember the answers to such questions, but that they should know how to find the answers.

Third Grade

In the third grade a good deal of your time will be taken with addition and subtraction and the large number of problems arising that call for these processes. You will be doing your best to get automatic response to the 100 basic facts in both. Do not forget, however, that you cannot push aside problems that will require multiplication and division. Some of these problems will arise in the natural course of events. Your classes will run across others, because you planned it that way. In order to deal successfully with such problems, your pupils should continue with the practice as suggested for grades 1 and 2.

Here is a possible problem: Tickets for the class play are 8 cents a piece. How many tickets can I buy for half a dollar?

Any one of the methods already mentioned can be used for a solution. If you allow your pupils to use their own initiative you may learn some worthwhile methods that have not occurred to you. The problem can also be solved by means of a table as below.[2] This is really a table of 8's.

After making this table, pupils can see that they can get 6 tickets but not 7. If they buy 6 tickets, they will get 2 cents change (2 cents will be left over). They should learn that the essential problem here is to find the number of 8's in 50.

As before, pupils should know that in the use of a table like this they have a *method* for solving all like problems. Note that division with a remainder is very little different from division without a remainder when these methods of approach are used.

As soon as pupils have learned to subtract, another method that comes a little closer to the more orthodox method of division is to start with 50 and subtract 8's in succession until 8 can no longer be subtracted. The pupils see that there are six 8's in 50 and 2 are left over just as they did by the other methods. (Incidentally we have here an excellent procedure for practice in subtraction.) If there is any question about the understanding here, the method can be associated with the drawing of circles as suggested in the first two grades. We have fifty circles and cut off 8 of them. Then we have 42 circles left, and so on. These numbers check with the subtraction method we have just suggested. Note that although the form of division is not being used the essence of the method, subtracting until the number is exhausted, is present.

50
8 ✓
42
8 ✓
34
8 ✓
26
8 ✓
18
8 ✓
10
8 ✓
2

The chances are your pupils will know at this time that $3 \times 8 = 24$. They can therefore buy three tickets at a time and the work will look like this. They are learning that it is more efficient to subtract 3×8 than to subtract 8 three times.

50	3
24	3
26	6
24	
2	

Sometime along in the spring of this third-grade year you will want to become more formal with division by such num-

[2]

No. of tickets	1	2	3	4	5	6	7	8	9	10
Total cost	8¢	16¢	24¢	32¢	40¢	48¢	56¢	64¢	72¢	80¢

bers as 5, 2, 3, and 4. You will probably introduce the symbolism such as $5\overline{)20}$ or $4\overline{)23}$ and work for automatic response to the division facts. When you introduce this way of writing division it is important for pupils to realize that it is not something new in concept but something new only in the way it is written and in the vocabulary used with it. Associate the new work you are doing with the previously learned methods. For instance, take an example like $4\overline{)23}$ and do it by all the methods the pupils have been using up to the present. It can be done more simply now because pupils know their tables of 4's and can see at a glance there are five 4's but not six 4's in 23. Do not present the division facts mechanically without understanding. Learning in advance of understanding cannot be permanent. Basic facts should in no case be learned before preliminary development of meaning. We do not want automatic response of facts taken on faith. We want automatic response of facts that have been discovered.

Fourth Grade

We spend a great deal of time, probably more than it is worth, on division in the fourth, fifth, and sixth grades. Our methods are quite mechanical. And because they are mechanical, we have to divide our examples into case after case; one place divisors with one- and two-place quotients, final zero in the quotient, zeros between other digits in the quotient, dividing by a two figure number, correct placement of the first quotient figure, to mention only a few. The method suggested here, because it is fundamental will make unnecessary some of these subdivisions. Uneven and even divisions need not be considered separately. Carrying or not has little significance. Zero difficulties can be made largely to disappear. Proper placing of the quotient figure is a meaningful procedure. Division by a two- or three-digit number is not greatly different from division by a one-digit number. What is more, there is a possibility of transfer of method from the simpler examples to the more difficult ones.

I was led to think about a different method because we spend too much time on division in the elementary school and because the results, although often good, may be completely meaningless. Granted that the methods I suggest are not as refined as the ones now used, I believe we have got away from the psychology which says, "Teach a process in the way it will be used." We have learned that the efficient refined methods of the adult should come after a period of slower cruder processes that are more within the children's experience.

Work in division in the fourth grade often begins with such examples as $2\overline{)62}$ or $3\overline{)189}$. Pupils learn how to do these examples readily enough, but I have found they do not know what their answers mean. I should prefer to begin with some such example as, "How many 8's are there in 67", assuming that pupils have not yet learned the table of 8's. Write it this way, if you like, $8\overline{)67}$. I would tackle it in any way suggested by the pupils, drawing 67 circles and marking them off by 8's or starting with 67 and subtracting 8's in succession. They will certainly know that $3\times8=24$ so they can save time by subtracting 3×8 instead of subtracting 8 three times in succession. (They can buy three tickets at a time, so to speak.) They will then see that 8 will "go" three times more and then two times. The work is shown at the right. The quotient is 8 and the remainder is 3. Whether the pupils do any better estimating of the quotient figure at this stage is immaterial. They are learning the essential process in division, subtracting from the dividend. They learn that if their estimate is too large, they must try again. If it is too small, all they have to do is to keep going. (Incidentally we allow this kind of thing in the reduction

```
  2   2
  3   3
  3   3
8)67  8
 24
 43
 24
 19
 16
  3
```

of fractions. 18/24, for instance may first be reduced to 9/12 and then the process continued.)

A sufficient number of examples on this level of difficulty should be given to insure understanding and then you might try "how many 19's are there in 126?" You find out whether your pupils understand what they are being taught by giving them, not examples like you have been working with, but examples that are just enough different to require transfer of ideas.

I think now I would graduate to two-figure quotients with one figure divisors but I would not take it as a different case, merely as a further development of the same kind of process. There is very little that is new. Take $8\overline{)126}$, for example. The first question is, "Will 8 go ten times?" (Note that multiplication by 10 is a prerequisite.) The answer is yes, so subtract 10×8. (Again bring in the idea of buying several tickets at once, if an explanation seems necessary.) Note that there is no question where the 1 in the 10 should be placed. It goes in tens' place. From there on the work is not new.

```
   5   5
  10  10
8)126  15
  80
  46
  40
   6
```

Probably by this time the pupils know the table of 8's and so will subtract 5×8 from the remainder 46. If instead, they do the work piecemeal, who am I to care? A further step is seen in an example like this: $6\overline{)157}$. The work at the right is self explanatory. Note that it is not a new type of divsion. It follows the pattern; it is merely a bit more complex. After it has been done, there is no law to prevent the teacher from showing how the estimate could have been better. It would have been simpler to have subtracted 20×6. And to help with the estimate, note that 6 goes into 15 twice.

```
   6   6
  10  10
  10  10
6)157  26
  60
  97
  60
  37
  36
   1
```

I now offer you another example of about the same level of difficulty. It has a zero in the quotient but nothing new has to be taught. 10×7 can be subtracted three times or 30×7 can be subtracted all at once. It is obvious that 7 cannot be subtracted from the remainder. The zero remains as it is.

```
  30
7)215
 210
   5
```

For 3-figure quotients, the same pattern can be followed. Examples are given at the right.

Take, for instance, $6\overline{)842}$. This time the question is, of course, "Does 6 go 100 times?" Again there is no question about the position of the 1. It must go in hundreds' place. Subtract 100×6. The remainder is 242. The divisior 6 will not go 100 times again. It is obvious that it will go more than 10 times. Pupils can now try *6 into 24* and subtract 40×6. Note that the zero difficulty again takes care of itself.

```
  40
 1ØØ
6)842
 600
 242
 240
   2
```

Here is another example with another kind of zero difficulty: $6\overline{)643}$. Does 6 go 100 times? Yes. Does it go 10 times in the remainder? No. It goes 7 times with the remainder 1. The 7 naturally goes in units' place leaving a zero in tens' place.

When the divisor is contained in the dividend two, three, four, etc. hundred times, you can develop the ability to estimate as you did with the tens.

```
   7
 1ØØ
6)643
 600
  43
  42
   1
```

Fifth Grade

In the fifth grade pupils are ready (if not before) for division with divisors of two or more digits. Even so it is not advisable to begin the new work before finding out how much of the development given in previous years has become a part of the pupils' mental equipment. Two general principles must be clear. (1) Division is a short way of making repeated subtractions with equal "subtractees,"

and (2) subtracting $a \times b$ gives the same remainder as subtracting b in succession a times. Do not hesitate to spend a little time drawing pictures. For example, divide 28 by 3 by drawing 28 circles and marking them off by 3's. Take the same example and subtract 3 as many times as possible. Then note that subtracting 9×3 is easier than subtracting 3 nine times and that the remainder is the same. Then go on for a brief time to larger numbers divided by a single digit following suggestions already made. "Does it go 100 times?" "Does it go 10 times?" you will remember are the key questions.

As soon as you are sure of this background, you can go on to an example like $32\overline{)129}$. Give the example without explanation and see how your pupils do it. You can expect it to be done by various methods from the crudest to the finest. Then discuss the example. You can subtract 32 four times in succession and have a remainder 1. Perhaps you would rather take it away twice and then twice again. It is obvious that the simplest way is to take 32 away four times at once; that is, subtract 4×32. In whatever way it is done, the result is 4 with remainder 1 and this means that 32 is contained in 129 four times with 1 left over.

Without being told, the chances are that pupils will realize that they cannot subtract 6×32 or 5×32. If they do need a discussion, be sure to give it. Multiplication will show in each of these two cases that the product is too large.

Practice with several examples of about this same difficulty.

At some time while these examples are being done, when you are sure that the meaning is clear, throw in the suggestion (and keep throwing it in until it takes) that there is an easy way to make an estimate of the quotient figure. But just to show the pupils that they can never be to sure of their estimate give this one before too long: $28\overline{)152}$. The only thing to do when the estimate is too large is to back up a bit. If you think your class can stand it, mention the fact that 28 is nearer 30 than to 20 and it would be better to try *3's into 15* than *2's into 15*.

It is now time to go on to two- and three-figure quotients. The pattern already suggested for the same thing with one-figure divisors can be used here. As a matter of fact the procedure is so much the same that the things I say will sound like mere repetition. Try an example like $21\overline{)324}$ without giving any help. It is likely that many of your pupils will get the correct quotient, showing that the process as a general procedure has been understood. Pupils should ask first, "Does 21 go 10 times?" (You will need to practice multiplying by 10 and 100 again.) Since the answer is "yes," subtract 10×21 and get a remainder 114. In placing the 10 in the quotient, the 1 of the 10 should, of course, go above the 2 because it must be in tens' place.

```
    5
   1Ø
21)324
   210
   114
   105
     9
```

We have now subtracted 21 ten times (10×21) and we see that the divisor will not go 10 times more. Try "2's into 11" and take 5 as a possible quotient figure. Subtract 5×21. Note that in all 15×21 has been subtracted. The quotient is 15 and the remainder is 9. Pupils should be led to form the habit of asking, "Does it go 10 times?" and in the larger numbers, "Does it go 100 times?" This is very important.

Note the example at the right: $23\overline{)489}$. Does 23 go ten times? Yes. Subtract 10×23 and get a remainder 259. Will 23 go ten times more? Yes. Subtract 230 again and get 29. You can now subtract 23 once more. You have subtracted 23 twenty one times and the remainder is 6. It is not difficult to show pupils how much easier it would have been to subtract 20×23 than to subtract 10×23 twice. Do not make this suggestion too soon. Let pupils do several examples the long way. It is

```
    1   1
   10  10
   10  10
23)489  21
  230
  259
  230
   29
   23
    6
```

likely that some of the pupils will suggest the shorter way without your help.

It is time now, of course, to practice on division examples in order to establish correct thinking and skill. Transition to examples like 32)3692 where there are three figures in the quotient should be easy. The key question is "Does 32 go 100 times?"

Note again that so-called zero difficulties take care of themselves. This is shown in the examples that follow.

```
  10          6
24)246      20Ø
 240      24)4961
   6       4800
           ----
            161
            144
            ---
             17
```

Such refinements as come in naturally may be made at any time but pupils should never lose sight of the fact that the essential part of division is the subtracting. "How many times is 24 contained in 4926?" is answered by finding out how many times 24 can be subtracted from 4961.

I have suggested this method as much more meaningful and less mechanical than the time-honored method. Teachers who have not tried it and have not thought about it carefully will say that it is confusing. It cannot be done *well* without the cooperation of teachers all along the line. For a fifth grade teacher to attempt it in its entirety when her pupils have had a year or two of the older method could be confusing. Even so I have seen a fifth grade teacher make use of some of the elements of this discussion to advantage while leading quickly to the refined type of division. You, as an individual teacher, can profit by getting the point of view I have tried to give even though it takes a change from the early grades to follow the method entirely as I have described it. In spite of its crude beginnings it leads through understandable stages to the more refined methods used by adults. I doubt if I would try to teach the method of long division as we know it now until pupils are ready for division with decimals. I would not care much if some of the pupils persisted in the less efficient methods even then.

A Puzzle

By Frank Hawthorne, Hofstra College, Hempstead, New York. All admissions to a certain basketball game were 50 cents. The mathematics teacher, who also sold tickets to games, provided himself with a quantity of half dollars, four dollar bills, a couple of fives and a ten. He was not able to make change for the first man who wanted to buy a ticket. Why not?

(*Answer:* The man offered a quarter and three dimes.)

What Constitutes Remedial Work in Arithmetic*

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THE concept of remedial arithmetic should undergo changes similar to those of remedial programs in other fields. In the medical field, the practice in former years was to treat the patient in order to give relief, hoping to effect a permanent cure. An abdominal pain was treated as a pain to be stopped as soon as possible, often with tragic results if the pain happened to be a symptom of an infected appendix. More and more doctors are relying upon clinical diagnosis to determine the real difficulty and to suggest the treatment. Only those temporary measures are used which are absolutely safe. As some one has said, "Before they dose, they diagnose." In the social field, juvenile delinquency was treated as a crime, and punishment meted out. Now we are asking: Why did he do this? What is the underlying cause? So, in remedial arithmetic work we can no longer merely treat the symptom, but must look carefully for the cause and take measures to bring about a permanent cure.

There are three phases of remedial work which should be considered. First, there is the individual who comes up as an isolated case and needs attention. Formerly this child was failed and required to repeat a grade. Perhaps (altho it is doubtful) this solved his arithmetic difficulty, but it created emotional problems of much more concern. It is now the general practice to keep him in his social group and to diagnose and treat his arithmetic ills.

It should be noted that the problem of the retarded student is not confined to the non-achiever. The above average student is often passed by because he knows the arithmetic of his grade level. Perhaps he is not working up to his ability, perhaps he dislikes arithmetic, or perhaps he cannot rely on his own resources to complete a task. Such a student needs remedial aid of a different kind than the non-achiever, but aid nevertheless. In this group are our future scientists, research people, and statisticians.

Second, several cases of arithmetic difficulties may arise, an epidemic, to use a medical term. While in such a situation the individual must still be treated, we must recognize that the trouble lies elsewhere, perhaps with the teacher, perhaps the school system. Such a condition should be met by first determining the underlying cause.

Third, there is the long range program of basic research and study, one which will prevent the occurrence of difficulties, if possible. We must start worrying before it happens. Such research is being carried on in the laboratories of our colleges of education and in our classrooms. It is the responsibility of the teacher to keep informed on these findings.

There are three points of re-education to be considered as we study a remedial program, the child, the teacher, and the parents. It is recognized that no educational program, remedial or otherwise can succeed without cooperation between the child, the school, and the home.

THE INDIVIDUAL CASE

Let us consider first the individual case. What are the necessary steps? We must first discover the deficiency. We may already know it. On the other hand, difficulty in arithmetic may show up in the form of a discipline case, and perhaps not always in the arithmetic class. Habitual tardiness or absence may be the result of trouble in the arithmetic class. Here again it is just as likely, perhaps more so, to be the bright student who is in trouble. He

* Presented at the meeting of the National Council of Teachers of Mathematics, Baltimore, April 1 and 2, 1949.

may be bored because his mentality is not challenged to put forth its best effort.

There are several ways of discovering the nature of the arithmetic ill. The old standby method was by a test. Formerly tests merely indicated that the pupil did not know a process, e.g. let us say long division. More recently, tests have become more diagnostic in nature and will reveal more. But in the last analysis there is nothing like a personal interview in which the pupil thinks out loud, or if time does not permit this, observation of his work, with a frequent, "Why are you doing that?" or "What are you doing now?" from the teacher. This paper deals with a more basic remedial problem than that of overcoming a computation difficulty, but such temporary ills must be taken care of.

Suppose a sixth grade child has worked and missed this problem:

$$\begin{array}{r} 206 \\ \hline 19)3921 \\ 38 \\ \hline 121 \\ 114 \\ \hline 7 \end{array}$$

Observation may indicate that the difficulty may lie in any one of a number of places, e.g.:

1. The division, multiplication or subtraction facts
2. Uneven division
3. Zero in the quotient
4. Two digit divisors where the trial quotient is not the correct one
5. Multiplication which involves both multiplying and adding
6. Multiplying with the multiplier not directly under the multiplicand
7. The process of division itself; why multiply? why subtract? etc.

A test composed of several parts, each testing one of the above processes, might have revealed where the difficulty would arise before the process was attempted. Correction of any or all of the difficulties involved here may help the individual. A pre-test before teaching will often save future trouble.

Such a diagnosis is sufficient if the difficulty is of temporary computational nature. If the difficulty is deep seated and if it is resulting in conflicting emotional reactions or if temporary relief does not cure, then a clinical diagnosis including other factors than arithmetic is necessary. Some possible discoveries in such a diagnosis are:

1. Low I.Q. Research has shown how much arithmetic is to be expected of the low I.Q. and these goals should be kept in mind. To place them too high results in frustration. A fourteen-year-old student whose tests indicate sixth grade achievement in arithmetic and sixth grade ability is not retarded. He is working to capacity and this fact should be kept in mind.

2. Short memory and attention span. It is particularly evident in long column addition or in multiplication involving four or more numbers. Keep the work simplified for students with this handicap. Such a deficiency accompanies a low I.Q. But it may also be the result of low vitality or of social immaturity. A child, because of illness or continued dependency, may not have developed his memory or attention span to its fullest capacity, and this may be one of the remedial measures necessary.

3. Reading difficulty. The correlation between reading and arithmetic is usually very high. In all cases the teacher of arithmetic should be a teacher of reading. There is a special reading skill involved in mathematics which is comparable to reading a passage, then translating into shorthand. An arithmetic problem involves reading, writing it in shorthand, then performing the operations indicated by these symbols. More emphasis should be placed upon teaching reading in the arithmetic class and upon arithmetic material in the reading class.

4. The pupil may not be ready for the work; his experience may be meager. Two

students on a fifth grade achievement level, for example, one ten years old and the other sixteen, are not ready for the same work in arithmetic, particularly in the social content field.

5. The pupil may be visual-minded and verbal explanations may disturb his thinking. Give the visual-minded student a quiet time in which to figure things out for himself. The audio-minded individual usually fares better in the average classroom.

6. An emotional block may be the cause. The entire time might be spent in discussing this phase alone. What are some causes of the emotional block? Non-success in arithmetic is a very frequent one. Arithmetic is a logical, sequential subject, and missing any step in this process will cause difficulty. Non-success, particularly if he sees others around him succeeding, will cause him to feel inferior. Competition may discourage him; it is not only unfair but futile. Only those children who are working up to their ability and who are experiencing success can develop into well adjusted, integrated personalities, which is after all the aim of education. We cannot escape the responsibility of trying to rid the public, the parents, and the child of the notion that arithmetic achievement is so important in his life that he needs to become frustrated over it. He may know less arithmetic, but what he knows he has mastered and can use, and he will be a happy, well adjusted individual.

Parents often push children beyond their ability, particularly if a brother or sister or the parents themselves have been exceptional students. Or, they inform the child that arithmetic is hard, that he may expect to have trouble: "I always have; arithmetic is hard." The author had this fact very forcibly brought home one time in a ninth grade math class. A youngster for six weeks had been doing very satisfactory work. His father came to visit and in the presence of the youngster said, "I don't expect ———— to do well in his algebra; I always had trouble with it and I am sure he will." From that time he did. Parents frequently discourage children by such statements as, "I can't understand why ———— has so much trouble, his older brother does so well." If for one generation we could keep the parents from making statements such as these, half of our arithmetic problems would be solved.

High I.Q. children may stop working in arithmetic because they achieve less there than in other subjects; they may have a perfectionist attitude and refuse to try. Perhaps to place such a child in the position of needing to know some arithmetic would motivate him to make the initial effort.

7. Arithmetic difficulties may be only a part of the trouble which the child is having in school. His low achievement may be the direct result of an unhappy home situation. Perhaps he did not have an adequate breakfast; perhaps the father came home drunk the night before. All of such possibilities should be examined. They are a part of the total picture.

8. Poor muscular coordination, perhaps due to tenseness over an arithmetic situation, may result in untidy papers and crooked columns. Such a child should be allowed to use larger muscles, perhaps at the board or an easel.

What is to be done after the seat of the difficulty has been discovered? A few general suggestions for corrective treatment can be made:

1. Do not place too much emphasis on the fault and let the child recognize, with you, his dilemma. He will be relieved to find out that someone knows about it and plans to do something besides penalize.

2. Start the child where he will immediately experience success and base the work wherever possible on his immediate needs. Every child, at any achievement level, is ready for some kind of arithmetic experiences. We must wait for the child to be ready to read, but even the little two-year-old who goes to the cookie jar and brings enough cookies for himself, his mother and father, is ready for the initial number experience of placing into one-to-one correspondence. He may not be on the level of the rest of the class, but he has a level of his own on which

he can succeed and which the teacher must find. "Nothing succeeds like success."

3. Examine carefully the child's basic concepts; his trouble may have started in the primary grades by introducing abstract numbers too early. You may need to begin at the lowest level and build up (through experience) the proper concepts.

4. Allow the child to discover the processes by himself. Perhaps we rely on verbal explanations too much. Give him plenty of time, and materials to work with, put him on his own, and he will get a thrill and a feeling of security when he makes a discovery.

5. Teach the child how to study arithmetic, how to read the problem and illustrate it if necessary, what tools to have, the importance of correct form and neatness, the necessity for continual self-evaluation.

In the directions accompanying the "Group Diagnostic Reading Aptitude and Achievement Tests," Marian Monroe gives eight principles of remedial work in reading which are equally applicable to remedial arithmetic and which summarize what has been said:

(1) Begin always with reading (arithmetic) material at the level at which the child can succeed.

(2) Build up confidence in ability to learn.

(3) Select reading (arithmetic) material in keeping with child's interests.

(4) Give generous praise for every slight success.

(5) Note special difficulties and arrange work which will help overcome those difficulties.

(6) Arrange progress charts and other ways of demonstrating improvement so that the child can recognize his progress.

(7) Avoid unfavorable comparison with children who are more capable.

(8) Retest after a few months to measure progress.

Group Difficulties

Experience has shown that in the case of group difficulty there are several factors which may be responsible, all of which are the earmarks of poor teaching:

1. The teacher may have insisted that there is a correct way to do a certain type of problem which may not be the easiest way for the child and the one which he will understand.

2. The instruction may have been aimed at the average (or even the above average) and pupils with difficulty get discouraged, and begin to have trouble.

3. As mentioned above, the abstract process may have been presented too early; or it may have been taught from a textbook instead of from real life situations.

There was the case (an actual one) of a little boy whose playmates were worried because he did not know the answer to 2+1. The help of the father of one of the playmates was solicited. He asked the child, "If you had a dime, how many ice cream cones could you buy?" His immediate reply was, "Two."

"If someone gave you a nickel, how many could you buy?"

"One."

"How many would you then have?"

"Three" was the immediate reply. This child saw no connection between the abstract 2+1=? and the concrete problem of finding how many cones he had.

The four stages of abstraction should constantly be kept in mind: the object stage, the picture stage, the semi-concrete stage, and the abstract number stage. It may be necessary in the remedial program to begin again at the object stage, and there is an object stage for almost any arithmetic process. It must be remembered that certain low I.Q.ers may never be able to reach the abstract stage. In a certain high school in our city two boys, brothers, were put into the same math class because the older (but duller) was a good influence on his younger brother, usually succeeding in getting him to class at least. They were allowed to sit at the back of the room and to work at their own speed. One day the teacher found them down on their knees on the floor with their arithmetic papers. Upon inquiry, the teacher was told by the brighter of the two, "I'm teaching ———— to work these problems by counting on his fingers." With a 56 I.Q. he probably will not get beyond the finger counting stage. But he is succeeding and will find in his own way how to do the arithmetic which will be adequate for him.

4. Too much emphasis may have been placed upon speed, upon getting the answer, neglecting the emphasis upon content and meaning.

5. The teacher may have failed to rec-

ognize that children learn through seeing, hearing, touching, speaking, handling. No avenue should be omitted.

6. Certain basic concepts may be lacking; they may have been taught by rule (perhaps by the parents). Abstract number may have been the starting place. A few of the most important concepts should be mentioned: the meaning of number, zero, our decimal system of number, the reasonableness of answers. You will agree that many, perhaps the majority, of the difficulties of computation may be traced back to a lack of understanding of the place value of number and the meaning of zero.

7. The group may not be ready for the new processes when presented. Diagnose before introducing any new subject in order to determine the strength of each student. On the basis of this diagnosis, clear up any difficulties before introducing the new material.

The Long Range Program

The long range program is really a prevention program, a problem to be attacked by teachers and administrators working together, and they in turn keeping the parents informed of what they are trying to accomplish. Prevention should be the ultimate goal. It represents the highest level of diagnosis.

1. Basic research is constantly being carried on and the results of such research are made available. No school should be without a professional library in which such materials are to be found.

2. Children learn arithmetic as they learn anything else, and the learning may affect their emotional stability. We owe it to them to get a better understanding of them and how they learn. Too much emphasis upon arithmetic and too little on the child has done more harm than we perhaps realize.

3. A constant evaluation of the arithmetic work within a school should be made. When we think of evaluation we usually think of standardized tests. They have their place. If a school system is far below the national norm in arithmetic achievement, it should be a matter of immediate concern, a recognition that there is something that needs to be remedied. But aside from this type of evaluation, ask yourself, ask those who meet the products of our schools, "Can they use arithmetic?" The army said that the boys could compute on paper, but could not use that power to solve a problem in the field. This, after all, is the real test of arithmetic teaching.

4. In the last analysis, corrective or remedial teaching is simply correct teaching. Correct teaching begins at the child's level and aims toward the highest level of accomplishment of which he is capable. Heavy class loads often make such teaching imperfect, but much more can be done than at present if we are constantly aware of it.

To summarize; a remedial program for the individual and the group and a prevention program involves finding a satisfactory answer to the following questions:

1. Who is having trouble? The individual, the group, or perhaps the teacher?

2. Where is the difficulty? Is it a surface difficulty which is quickly cured or is clinical diagnosis necessary, both of the pupil and of our philosophy of education and the place of arithmetic in education?

3. Why did the trouble occur? Does the diagnosis indicate it is an emotional, mathematical, or, on our part, an educational one?

4. What remedies are suggested for the child, for the group, for prevention? Are these remedies giving temporary relief or do they cure the basic fault? If only temporary, are we sure that it is not aggravating the underlying cause? Do we still use as our remedial procedure: finding what he does not know, showing him how to do it, and then drilling until he won't forget it? (Or at least we hope he won't.)

5. Whose responsibility is it? Is the school meeting its responsibility, or has the school in its interest in remedial read-

ing and social living forgotten that the child's number experiences are a basic part of both?

In medicine, preventive and corrective measures are not entirely successful, but they do lessen the physical suffering and fatalities. So, in arithmetic, prevention through research and study and a remedial program for group and individual difficulties will, through cooperation between the home, the school, and the child, save much unhappiness in the arithmetic classroom. We will succeed, not in sending all pupils to high school equally prepared mathematically, but in sending them prepared to the highest degree of their ability and with a maximum degree of confidence in what they are able to do.

Algebraic ABC's

By PAULINE WILSON
James Hillman Jr. H. S., Youngstown, Ohio

(A parody to the recent popular tune, "A—You're Adorable": The Alphabet Song)

Verse:
While Johnny Jones was working on his problems
He fell asleep and dreamed of x, y, z's;
All the letters left their stations,
Came out from equations,
And made Johnny say again his ABC's.

Chorus:
A stands for algebra, B for binomial,
C for coordinates, and then
D stands for digits and E for equations,
And F means to factor (if you can);
G means the graphs we use, H the hypotenuse,
I—some irrationals we find.
J is for Job who had problems almost as bad
As those we pupils are assigned.
K—L—M—*N*, numbers before our eyes,
O—P—Q, then, algebraic'ly R for rationalize.
*S—T—U—*V*, vocabulary woes,
*W means we have to work and Y means this year soon goes;
Z's Mr. Zero who will no dividing do.
One's still a mystery . . .
Yes, X is missing here, when in my mind it's clear
I'll tell you what it means to me.

(Repeat ending—if desired)
Yes, if I should find out what X is all about
I'll tell you what it means to me.

* Music for preceding two lines must be repeated for these two.

The Content of a Course in General Mathematics —Teachers' Opinions*

By KENNETH E. BROWN
University of Tennessee, Knoxville, Tenn.

MY INTEREST in general mathematics was stimulated several years ago when I attempted to teach general mathematics to seventy girls who, expecting to be primary teachers, saw no need for mathematics. Since that time I have given special attention to the attempts of other teachers to fill the mathematical needs of that large academic group who are not specializing in mathematics or its applied fields.

THE SURVEY IN 1942

In 1942, after analyzing more than fifty textbooks in general mathematics, 458 questionnaire answers from colleges and the opinions of 1500 students, I reached the following conclusions: (*General Mathematics in American Colleges*, Bureau of Publications, Teachers College, Columbia University.)

1. According to the authors of the texts and the teachers of the subject, general mathematics could be classified in three divisions based on the objectives of the course as follows:
   a. Preparatory: to equip students for a profession, semi-profession, or a vocation in which mathematics is a useful tool.
   b. Cultural: to prepare students to be intelligent citizens mathematically, to transmit those skills and concepts that are the necessary prerequisites of a desirable citizen, to emphasize the understandings of mathematical concepts, and to show the relation of mathematics to the other great fields of learning—all with little emphasis on the manipulative process in problem solving.
   c. Combined (Preparatory and Cultural): to attain both the above objectives by meeting the needs of the large academic terminal mathematics group and likewise furnish adequate preparation for the minority who wish to pursue further courses in mathematics.
2. The preparatory type of general mathematics seemed to be in the most stable position. In comparison with the other two types of courses, preparatory general mathematics had been offered a greater number of years and it was rated higher by both teacher and student.
3. The combination course was ranked highest in disfavor by both teacher and student. There seemed to be little success in accomplishing both objectives except with some classes of superior students.
4. The cultural course had only recently made its way into the colleges. Nearly half the colleges answering the questionnaire had offered cultural general mathematics for a period of less than four years. Considering the number of colleges that had recently introduced the course, there seemed to be a trend toward general mathematics for the terminal student. It was pointed out in that report that this trend might not continue since the national crisis might cause the students to elect the traditional courses.

THE SURVEY OF 1947

In 1947 another questionnaire was sent to 480 colleges whose catalogues indicated

* Read at the twenty-seventh annual meeting of The National Council of Teachers of Mathematics at Baltimore, Maryland on April 1 and 2, 1949.

that they offered a course in general mathematics. The questionnaire answers from these colleges indicated that 40% of the colleges which had offered a course in general mathematics for the cultural student had discontinued the course. The reasons for dropping the course were given as follows:

1. The textbook was unsatisfactory.
2. Because of increased enrollments in college since the war, the overloaded mathematics faculty could not offer both the cultural course and those designed for the specialist in mathematics.
3. During the war, the college enrollment dropped and the students' interests were turned toward traditional mathematics.
4. Students could change to a mathematics major without loss of credit if the traditional courses were offered.

Thus the questionnaire in 1947 disclosed that general mathematics for the cultural student had been curtailed by the lack of proper teaching staff in the face of increased enrollments and the lack of a desirable textbook.

The Survey of 1949

Since the teachers had expressed dissatisfaction with the textbooks and now that the great influx of veterans is beginning to wane, it was decided to pursue the investigation further along the following lines:

1. What topics do the teachers consider important in a general mathematics course for the cultural student?
2. Now that teaching staffs are more readily available, is there a trend toward general mathematics?

In March 1949 a questionnaire was sent to the universities, colleges, teachers colleges and junior colleges for white students listed in the Educational Directory of United States Office of Education. More than one thousand questionnaires were returned by these institutions. Approximately one hundred returned questionnaires stated that the institution did not offer a course in general mathematics and the teacher was undecided as to the desirable content of such a course. However, more than nine hundred institutions returned questionnaires that indicated the topics considered important in a cultural course in general mathematics.

Topics Considered Important

Arithmetic and consumer problems were selected by 90% of the teachers as material that should be included in a cultural general mathematics course. In fact, 42% of the teachers would place "great emphasis" on the material selected from the field of arithmetic. (See Table 1.) Typical of the comments from this group of teachers is the following: "Most students irrespective of the extent their secondary background can well afford to spend considerable time on the study of arithmetic. This should come at the first of the course." This point of view has been expressed by C. V. Newson in the following words: "It is my firm belief that any successful course in general mathematics must start with a study of arithmetic." ("A Course in College Mathematics for a Program of General Education," *Mathematics Teacher*, Vol. XLII, No. 1, Jan. 1949. p. 20)

There seems to be a difference of opinion among the teachers in regard to the importance of mental arithmetic. Approximately one-fifth of the teachers indicated that they would not include it in a cultural general mathematics course while the same number of teachers would give it great emphasis. From the many notes written by teachers explaining the meaning of mental arithmetic it would seem that the apparent differences of opinions are due largely to the interpretation of the meaning of the term "mental arithmetic." Usually those teachers who indicated the desirability of including the topic implied that mental arithmetic is the

estimation of an answer or a mental short cut in arriving at the answer. Those teachers who did not think it should be emphasized usually indicated that mental arithmetic is the same procedure in arithmetic except without pencil and paper. From a study of the comments on the questionnaires, it would seem that many persons who would not include mental arithmetic as they interpreted it would emphasize the ability to estimate answers and certain mental short cuts in approximating answers.

On no topic did so many teachers agree that it should be included in a course in cultural general mathematics as the topic of algebra. Of the 900 questionnaire answers, 92% indicated that material from algebra should be included in the course. 27% of these teachers stated that it should receive "great emphasis." The following comment indicates the opinion of one of the teachers in this group: "We can ill afford to be ignorant of at least the basic course—algebra. The best encouragement my students get to take algebra does not come from those students who have interests in mathematics and the applied fields, but it comes from those students who have gone into graduate work in supposedly unrelated fields and have run into difficulty because of algebraic implications. I would almost go so far as to say to the student who graduates from college without a course in algebra that for his benefit there should be placed above the exit of the building a sign reading, 'Let no one leave these halls permanently without a knowledge of algebra.' "

According to the teachers in this survey, geometry, trigonometry and statistics also should have a place in the cultural course but with less emphasis than the material selected from algebra. Analytic geometry and the calculus were not so popular. In fact, 35% of the teachers checking this item indicated that the calculus should have no place in a cultural course of general mathematics. (See Table 1.) The topics selected from advanced mathematics were not considered desirable for a cultural general mathematics course by more than half of those expressing their opinions.

TABLE 1

The Opinions of 900 Teachers in Regard to the Importance of Certain Topics in a Cultural General Mathematics Course

Interpretation of the table:

The number in each column is the percent of the teachers who checked that column. The check meant that material from that topic should receive, (column 1) no consideration, (column 2) little emphasis, (column 3) average emphasis, and (column 4) great emphasis, in a course in cultural general mathematics. Example, the importance of arithmetic in a cultural general mathematics course was rated by the teachers as follows: 10% of the teachers indicated arithmetic should receive no consideration, 18% of the teachers indicated that arithmetic should receive "little consideration," 30% of the teachers stated it should receive "average emphasis" while 42% of the teachers indicated it should receive "great emphasis."

	1	2	3	4		1	2	3	4
Arithmetic	10	18	30	42	Anal. Geometry	22	37	33	8
Mental Arith.	22	21	35	22	Trigonometry	9	32	46	13
Algebra	2	14	56	27	The Calculus	35	36	22	7
Geometry	4	31	54	11	Topics from Adv. Math.	54	30	12	4
Statistics	8	30	46	16	Consumer Problems (Budgets, Insurance, etc.)	10	16	33	40

ADDITIONAL IMPORTANT TOPICS

The questionnaire requested the listing of topics other than those already mentioned that are "very important" in a cultural course in general mathematics. The order of the following topics indicate the frequency with which they appeared

on the returned questionnaires: history of mathematics, number system, mathematical reasoning, meaning of the processes, nature of mathematics, logic, uses of mathematics, appreciation of mathematics, field work, slide rule, empirical formulas and recreational problems.

Comparison of Opinions of Different Groups

It might be of interest to compare the opinions expressed on the questionnaires from institutions which offer cultural general mathematics with the opinions of the teachers of the entire survey. As Table 2 indicates, there is little difference in the opinions of the two groups.

Several of the returned questionnaires indicated, either by a comment or the textbook used, that the course was general mathematics only in name. In fact, the following comment is typical of this group: "As you can see by the textbook, the course we offer is not really general mathematics." The responses from this group were compared with the total group. The uniformity of the groups is shown in Table 3. Thus the opinions of the teachers in regard to the importance of the different topics for a cultural course in general mathematics were similar irrespective of the offerings of the college or the textbook being used. Material from arithmetic, algebra, consumer problems, trigonometry, statistics and geometry were considered important for the cultural student by all groups of teachers. Thus, analytic geometry, the calculus and topics from advanced mathematics were not so popular.

Method of Presentation

It should not be assumed that a weak solution of the traditional courses in elementary mathematics will fill the needs of the cultural students according to these teachers. It is neither feasible nor desirable to quote from all the teachers who commented on this point, but a few direct quotations will show their strong feeling in favor of proper presentation of fundamental ideas rather than manipulative exercises from selected topics of traditional material.

"The topics are not as important as the way they are done."

"Our ideas are that such a course should not emphasize mathematical techniques."

"It isn't so much the matter of emphasis but rather the view point."

Table 2

A Comparison of the Opinions of the Teachers of Cultural General Mathematics with the Opinions of All Teachers in the Survey

Interpretation of the table:

The first number (the number *not* italicized) represents the per cent of the teachers of cultural general mathematics who checked that column. The italicized number represents the per cent of the teachers of the entire survey who checked that column. Example: 35% of the returns from colleges offering general mathematics indicated that arithmetic should receive great emphasis as compared with 42% for the entire survey.

	1	2	3	4		1	2	3	4
Arithmetic	12 *10*	18 *18*	35 *30*	35 *42*	Anal. Geometry	20 *22*	38 *37*	35 *33*	7 *8*
Mental Arith.	32 *22*	22 *21*	34 *35*	12 *22*	Trigonometry	7 *9*	32 *32*	48 *46*	13 *13*
Algebra	3 *2*	12 *14*	58 *56*	27 *27*	The Calculus	36 *35*	38 *36*	23 *22*	3 *7*
Geometry	4 *4*	32 *31*	54 *54*	10 *11*	Topics from Adv. Math.	54 *54*	29 *30*	13 *12*	4 *4*
Statistics	8 *8*	26 *30*	53 *46*	13 *16*	Consumer Problems (Budgers, Insurance, etc.)	11 *10*	16 *16*	35 *33*	38 *40*

TABLE 3

A Comparison of the Opinions of the Teachers of General Mathematics (in Name Only) with the Opinions of All Teachers in the Survey

Interpretation of the table:

The first number (the number *not* italicized) represents the per cent of the teachers of general mathematics (in name only) who checked that column. The italicized number represents the per cent of the teachers of the entire survey who checked that column. Example: 36% of the returns from the colleges offering general mathematics (in name only) indicated that arithmetic should receive great emphasis as compared with the 42% for the entire survey.

	1	2	3	4		1	2	3	4
Arithmetic	16 *10*	18 *18*	30 *30*	36 *42*	Anal. Geometry	23 *22*	37 *37*	31 *33*	9 *8*
Mental Arith.	24 *22*	20 *21*	36 *35*	20 *22*	Trigonometry	10 *9*	32 *32*	44 *46*	14 *13*
Algebra	1 *2*	16 *14*	55 *56*	28 *27*	The Calculus	34 *35*	33 *36*	22 *22*	11 *7*
Geometry	4 *4*	31 *31*	53 *54*	12 *11*	Topics from Adv. Math.	53 *54*	32 *30*	12 *12*	3 *4*
Statistics	8 *8*	34 *30*	44 *46*	14 *16*	Consumer Problems (Budgets, Insurance, etc.)	12 *10*	18 *16*	32 *33*	38 *40*

"The topics do not tell the story. It is the way the ideas are presented that counts."

"I think that a cultural course in mathematics has most to offer in the way of fundamental concepts and methodology rather than specific content and problem solving."

"Drill for the sake of drill is worse than useless for the technical skills attained (if any) is soon forgotten but the bad taste in the mouth that accompanied the drudgery is not. To summarize, I would rather see a course taught in mathematics appreciation than a one-semester miniature of a four year formal course."

Thus the teachers state their desire for a vital course, with less emphasis on the development of technical skills, presented in manner that will appeal to the student.

STATUS OF GENERAL MATHEMATICS

The survey disclosed that 40 colleges expect to introduce a course in general mathematics next year and twenty-five other colleges are considering it among their future offerings. Although several colleges expect to add a course in general mathematics and 33% of the questionnaires from schools which offer the subject indicated satisfaction with the course, cultural general mathematics is in an unstable position. 30% of the colleges offering the course expressed dissatisfaction with the textbook. Comments on some of the questionnaires indicated strong opposition to the cultural course. The following comment expresses this point of view: "Honestly, I don't consider such a course desirable *at all*. I believe that a year of regular freshman mathematics will be of more cultural and practical value." However, there are many other teachers who would agree with the instructor who wrote this favorable comment: "I am enthusiastically in favor of the course." In fact, three-fourths of the teachers expressed a favorable attitude toward the course if a proper textbook could be found.

SUMMARY

According to the information received from 900 colleges, arithmetic, algebra and consumer problems should be emphasized in a general mathematics course for the student who does not expect to major in mathematics or its applied fields. Also trigonometry, geometry and statistics were considered important for the cultural student. Analytic geometry, the calculus, and topics from advanced mathematics were of less importance according

to their opinion. Other topics suggested as very important in a cultural general mathematics course were: history of mathematics, number system, mathematical reasoning, meaning of the processes, nature of mathematics, logic, uses of mathematics in civilization, approximate computation, business mathematics, appreciation of mathematics, field work, slide rule, empirical formulas and recreational problems.

The survey revealed that forty colleges plan to introduce a course in general mathematics next year. The cause of this trend, according to the comments on the questionnaires, is due to a decrease in the enrollment of veterans and an increase in the number of qualified instructors.

The great interest in general mathematics was shown by the teachers prompt indication of the topics deemed desirable for such a course—900 questionnaires were returned in two weeks. (The response of the colleges to all the questionnaires has been most remarkable.) This interest in general mathematics was expressed also by the many comments on the returned questionnaires. These comments expressed the desire to make mathematics meaningful to the students rather than present a maze of symbols that were of interest only to the instructor. Many teachers seem to be following the admonition of W. D. Reeve, a frontier thinker in the field of general mathematics for the secondary school, when he stated, "If more emphasis were placed on meaning and understanding and less on the mechanical aspects, pupils might obtain more value from the study of mathematics." (*Mathematics Teacher*, Vol. XXXIX, No. 3, March 1946, p. 134.)

Geometric Models Kit

Since the supply of the Geometric Models Unit is nearing exhaustion, teachers who wish to purchase this kit should place their orders soon. Many teachers have ordered the kits in quantities so that each student may have one. Other teachers have found the small models contained in the kit to be ideal patterns to use in the construction of large demonstration models. Directions for the construction of such permanent demonstration models are included with the kit.

The kit contains materials for making four small dynamic models which can be used to illustrate 25 theorems or ideas from plane geometry. An explanatory booklet is included. The kit was produced jointly by the National Council of Teachers of Mathematics and Science Service.

Orders should be sent to the chairman of the kit committee, Professor M. H. Ahrendt, Anderson College, Anderson, Indiana. The kits sell for 50¢ each or 3 for $1.00.

Have You Heard?

By Edith Woolsey
Sanford Junior High School, Minneapolis, Minn.

The twenty-eighth annual meeting of the National Council of Teachers of Mathematics is to be in Chicago April 12–15, 1950, at the Congress Hotel. From what we hear about it, this will be the biggest and best convention the council has had. The complete program will be published later, but there are some things you should know about it now.

The meeting is to be in the week following Easter. By then the weather should be right for an automobile trip into Chicago, unless you prefer another type of travel. The Congress Hotel is on Michigan Boulevard and Congress Street, overlooking Grant Park. Make your reservations with the hotel as soon as possible.

Room rates are as follows:

Singles: $4.00-$5.00-$6.00-$7.00
Twins: $7.50-$8.00-$9.00-$10.00
Three persons in a room at $9.00 per room
Four persons in a room at $10.00 per room

The Committee on Instructional Aids is very anxious to have an exhibit that will be of value to everyone who attends the convention. You have some device that has proved valuable in your work. Plan on putting it in the exhibit. Any communications about this should be addressed to

Mr. Reino M. Takala
Hinsdale Township High School
Hinsdale, Illinois

There will also be commercial exhibits. If you are interested in making reservations for commercial space, write to

Mr. E. L. Paine
Downers Grove Community High School
Downers Grove, Illinois

The convention opens on Wednesday with meetings throughout the day for the Board of Directors. Thursday morning there will be a meeting of delegates from state councils and affiliated groups. Be sure that your group is represented there. In the afternoon there will be a tour of Chicago museums, universities, and industries. This will probably include the Museum of Science and Industry, the Adler Planetarium, The University of Chicago, the Illinois Institute of Technology, and the Chicago campus of Northwestern University. Arrangements are being made for those who want to visit schools in Chicago, both on Wednesday and Thursday.

Thursday evening there will be two general meetings, one in each of the two large ballrooms. These will be followed by a reception. Friday and Saturday mornings at 9.15 there will be general sessions followed by various sectional meetings, group discussions, and panel sessions. You know, from other programs, what a wide variety of interests always appear in the topics chosen for discussions. You will have to attend the ones that are of greatest value to you, and get your friends to tell you what was said at the others. Several times each day the latest in mathematical films will be presented.

The time of the banquet has been changed from Saturday night to Friday night. This will make it possible for all of you to stay for the entire program and not have to rush off before the best speaker has finished, in order to catch a train. The banquet speakers will be Superintendent Herold Hunt from Chicago, and Dr. Everett S. Lee of the General Electric Company in Schenectady, N. Y.

Any luncheons? Yes, there will be two, the Get Acquainted Luncheon on Friday and the All States and Affiliated Groups Luncheon on Saturday. After the banquet there will be a Night (H)Owl Session. This

is the way it is described in the program: "Informal 'Coke' hour(s) in which new problems are expounded, actions lead to reactions, and discussions of functions of more than one variable result in new maxim(um)s." It sounds like history in the making.

You are wondering who is responsible for such an outstanding program. Our president, Dr. E. H. C. Hildebrandt, is the able leader to whom much of the credit is due, but he has had excellent assistance. Three affiliated groups are hosts for this meeting: The Women's Mathematics Club of Chicago and Vicinity, The Chicago Elementary Teachers' Mathematics Club, and The Men's Mathematics Club—Chicago and Metropolitan Area. You know, The Men's Mathematics Club is recognized as the official founder of the National Council.

Fifteen committees of teachers and principals from Chicago and neighboring cities have worked with endless energy and enthusiasm to plan this convention for you.

Chicago is centrally located and can be reached conveniently from all directions. April is a good time to be there. See you in Chicago at the Congress.

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Group Affiliation with the National Council

By H. W. Charlesworth

Chairman of Affiliated Groups, East High School, Denver, Colorado

Renewals of affiliation should have been in by December 1. By March 1 many new affiliations will be completed. Each of these groups will send one delegate to the Delegate Assembly in Chicago next April. Questionnaires on affiliation which have been sent to affiliated groups, to Board members, and to state representatives should be in by now.

The Chicago Delegate Assembly

This being a new venture for the National Council, we have no pattern to follow. It would be well for all of us to give it some good thought beforehand. To be successful, it must operate in a truly democratic way. The procedure outlined herein is suggestive of how it might operate. I shall appreciate having your reactions and suggestions for the improvement of the plan.

About February 15 I shall send letters to the affiliated groups asking for names of official delegates and requesting brief statements of questions that the delegate will raise at the Delegate Assembly. I shall compile these and make a summary report to the Delegate Assembly. This, together with the results of the questionnaire, will determine the agenda for the meetings. If possible, this agenda will be sent to each delegate prior to the Chicago meeting, in time for the delegate to refer back to his group for further information and reactions.

The Delegate Assembly will come to agreements on as many questions as possible. Decisions on minor matters will be readily reached. Those matters not so readily agreed upon will be referred to committees for more careful consideration. Since it is planned to have two sessions of the Delegate Assembly, these committees could meet in the interim and report back to the second session. Then, if agreement is not reached after a reasonable amount of discussion, the matter could be referred to the Committee on Affiliated Groups. This committee would take the matter under advisement, get Board action on it as soon as possible or submit it back to the Delegate Assembly the following year.

All decisions and recommendations of the Delegate Assembly must get the approval of the Board of Directors before they are put into effect. It should be possible to get many recommendations approved by the Board at the time of the convention. Since the Board will meet several times during the convention, reports to them could be made immediately. Those recommendations approved by the Board could be incorporated in the reports of delegates to their respective groups.

In addition to concerning itself with matters of affiliation, it is hoped that the Delegate Assembly will eventually become a policy forming body. It is quite probable, however, that the entire time of the Chicago Delegate Assembly will be consumed in shaping plans for better affiliation, this being the main purpose of it. Should the Delegate Assembly become an annual affair, it could develop into a very helpful policy forming group.

The first session of the Chicago Delegate Assembly will be at least two hours, the second about one hour. This means that we must operate expeditiously. It means that discussions must be held strictly to matters set forth in the agenda. It means that some important matters will not be settled. It means that delegates should be informed beforehand as to matters to be considered. It will be important to have in writing all decisions and recommendations which will later be presented to the Board. It will be necessary to have a stenographic record of the meetings. A complete report

of the proceedings of the Delegate Assembly should be made, as soon as possible, to the Board of Directors, to the affiliated groups, and to all members.

Your Delegate

As I have stated in previous articles, your delegate should be a member of The National Council of Teachers of Mathematics. He (or she) should be thoroughly familiar with the activities of the group and know its desires. And, of course, your delegate should be sincerely interested in affiliation and what it may mean to your group and to the National Council.

Naturally, the financial aspect of sending a delegate is important. The National Council has never been able to help in this. Many devoted people have been coming to conventions entirely at their own expense. Others have been more fortunate. It has been the practice of colleges to give financial help to members of the staff who attend conventions. Many school systems, even some of the smaller ones, now give help to teachers who attend conventions, some going as far as to pay all travelling expenses and subsistence in addition to giving time off with pay. Some teachers may not know that their school will help and have not received this help merely because they have failed to apply for it. It would be well to inquire into this possibility. While it may be easy to find the right person to represent your group, that person may need some financial help. Many groups use money collected from local dues for the purpose. In any event, we hope that every affiliated group can be represented at the Chicago Delegate Assembly.

I shall ask for a position at the registration table at the Chicago convention, thereby making myself available to any who wish to get in touch with me. I want to talk with you about your group and about affiliation matters. I wish to talk with those who may be interested in organizing groups to affiliate with the National Council. I want to meet personally all official delegates before the first session of the Delegate Assembly. I expect to devote most of my time while at the convention to matters of affiliation. So, please look me up and let's get acquainted. I shall welcome any suggestions you may have. Write to me now and see me at the convention.

New Affiliations

Groups desiring affiliation with the National Council will please write to H. W. Charlesworth, East High School, Denver 6, Colorado. If affiliation is completed by March 1, the group will be entitled to send a delegate to the Chicago Delegate Assembly.

REMEMBER THE MARCH OF DIMES!

Miscellanea—Mathematical, Historical, Pedagogical

By PHILLIP S. JONES
University of Michigan, Ann Arbor, Mich.

THE SCIENCE TALENT SEARCH

ARE you helping your students to "miss a boat"? You are if you have not realized the possibilities offered to your students by the Westinghouse Science Scholarships. The forty final contestants for the $11,000 worth of scholarships donated annually by the Westinghouse Electric Corporation are determined through a Science Talent Search administered by Science Service. One of the requirements for each competitor is that he write an essay on "My Scientific Project." Four of the projects submitted by the 1949 finalists dealt with mathematical topics. The topics were: "Geometrical Properties of Equations in Different Systems of Coordinates," "Three Four, and Higher Dimensional Determinants," "Mathematical Reflections Through Various Curves," "The Integraph and a Series of Graphs to Illustrate Its Action." The authors were respectively Norman Bauman (16), of Midwood High School, Brooklyn, New York; Robert Blattner (17) of Shorewood High School, Shorewood, Wisconsin; Alan Goldman (17), Abraham Lincoln High School, Brooklyn; Henry Landau (18), Bronx High School of Science.

Caroline Stuart Littlejohn, 16, of Classen High School, Oklahoma City, Oklahoma, submitted an essay "Beginning Researches in the Mathematical Theory of Probability" in addition to a non-mathematical project. She won a Grand Science Scholarship.

In 1948 five of the finalists' projects were mathematical and in 1947 four. Five of these nine winners were from the New York metropolitan area. Eugene, Oregon; Cliffside Park, New Jersey; Los Angeles, California; and St. Louis, Missouri were also represented.

Mathematics teachers should consider this contest and its projects as stimuli for old or new mathematics clubs, as extra club and class tasks for superior students, as opportunities which it is their duty to call to the attention of their better students. Don't neglect the possibility of having mathematical programs and projects in existing science clubs if you don't have a separate mathematics club.

More information, club program suggestions, reading lists, and suggested projects are all to be found in the free literature and *Sponsor's Handbook* distributed for the Science Clubs of America by Science Service, 1719 N Street, Northwest, Washington 6, D. C.

In many states there is a statewide Junior Academy of Science with publications, materials, and meetings which help a club sponsor plan a program and stimulate interest.

OLD PROBLEM—NEW DOGGEREL

Professor Norman Anning of the University of Michigan gives us the following note:—

F. Hirt of Breslau has published for Heinrich Doerrie two books of solved problems: *Triumph der mathematik* in 1933, *Mathematische miniaturen* in 1943 Here is a translation of problem 260 of the second set

> A man is twice as old as was his frau
> When he was just as old as she is now.
> And, if they both survive until the frau
> Will get to be as old as he is now,
> Their ages then will total 99.
> Come on. Lay out the ages on a line.

The problem *How old is Anne?* has appeared several times in these pages. A problem to end all such problems was that of the hippopotamus's child; it was in the *Mathematical Gazette* in or near 1923.

MATHEMATICAL INSTRUMENTS

Lauren G. Woodby of the Canal Zone Junior College writes: "The Canal Zone Schools recently received a valuable assortment of equipment for instructional use in mathematics and engineering. Through the cooperation of the United

States Army, Caribbean, several thousand dollars worth of supplies—including pantographs, slide rules, calculators, parallel rules, compasses, planimeters, protractors, clinometers, alidades, transits, levels, and theodolites—were made available to the Schools Division of the Panama Canal."

"Every freshman engineering student in the Canal Zone Junior College is now furnished with a slide rule on a loan basis. Descriptive geometry students are loaned a parallel rule, engineer's scale, and a large nickel silver protractor with vernier arm. Sufficient instruments are now available for the second semester (dry season) surveying course so that a class of twenty students can work in parties of two without the necessity of staggering the field work."

This reminds the editor that some war surplus instruments are still available and that mathematics teachers might well use the economy of such purchases as an argument for finally getting some money for mathematical instruments and supplies. Administrators can be sold the idea that mathematics too needs equipment. The editor purchased a war surplus parallel ruler for half price ($1.49) last week. Mr. Cook of the Muskegon (Michigan) High School was able to obtain funds for an oscilloscope which he uses for demonstrating sine waves in trigonometry as well as in physics.

Mathematics Scholarships for College Freshmen

The report on contests and scholarships in mathematics in THE MATHEMATICS TEACHER for October, 1949, included, on page 284, references to scholarships in mathematics for first year college students. More detailed announcements of these scholarships are reported in the following paragraphs. Teachers should write directly, and without delay, to these colleges if they have promising students in mathematics who are seeking financial assistance for college work.

It is to be regretted that there are not more colleges and universities offering awards to new students who can demonstrate special promise in mathematics. If attention of college people is called to these contests through this section of THE MATHEMATICS TEACHER, and by interested teachers and teacher organizations the number of opportunities of this kind might be considerably increased.

HOFSTRA COLLEGE, HEMSTEAD, NEW YORK

Competitive examinations are given to entering freshmen in the spring of the year before they enter the college. Applicants for these scholarships are required to take a three hour general aptitude test in the morning and a one hour test in mathematics, made out by the Mathematics Department, in the afternoon.

These tests are devised to find out not what a student has memorized but to what use he can put the mathematics he has learned. Although, there are no trick questions asked, the applicants are required to use their abilities on types of problems which are new to them and which require the ability to think.

The awards for these scholarships amount to $250 a year and are renewable each year as long as a student maintains an over-all average of 85 per cent.

KENYON COLLEGE, GAMBIER, OHIO

Kenyon College offers four-year scholarships to qualified men who do well on an examination in a secondary school subject of their choice. Contingent upon need, the amounts of Kenyon scholarship awards are $5000 for George F. Baker Scholarships, $4000 for Procter National Scholarships, $3000 for Kenyon Prize Scholarships, and $2400 for Kenyon General Scholarships. The scholarship examinations are designed to give the candidate an opportunity to show what he can do with the subject rather than simply to test his knowledge or ignorance of facts. The examination in mathematics while presup-

posing a knowledge of Algebra and Plane Geometry is primarily designed to measure the candidate's ability to reason carefully and to give precise mathematical demonstrations and proof. Scholarship winners are not required to go on in the subject in which they took their examination. Further information about Kenyon scholarships may be obtained from the Director of Scholarships, Kenyon College, Gambier, Ohio.

Mr. Donald Du Bois, Kenyon College, Director of Scholarships, reports that of the one hundred and eighty-two men who wrote scholarship examinations last year at Kenyon College, fifty-four wrote in the field of mathematics. Of these, ten were awarded and accepted scholarships and are now in the freshman class.

Rockford College, Rockford Illnois

Rockford College, a small liberal arts college for women, offers Departmental Scholarships of $600 ($300 each year for two years) on the basis of competitive examination, high school record and recommendations, score on aptitude test, and other pertinent information.

A Departmental Scholarship in mathematics is offered each year. In order to compete for this scholarship, a student must rank in the upper fifth of her class in secondary school and must have completed one and one-half years of algebra and one year of geometry. The scholarship examination may include questions on any course in mathematics completed prior to the date of examination.

All applications for Departmental Scholarships must be filed by March 1. A student must have been accepted by the Committee on Admission of Rockford College before her application for a scholarship can be considered. Admission forms may be obtained by writing to the Director of Admission, Rockford College, Rockford, Illinois.

Stanford University, Stanford California

The Stanford University competitive Examination in Mathematics was given in April, 1949 for the fourth time. Professor G. Szegö, executive head of the Department of Mathematics of Stanford reports that his department is fully satisfied with the results of and with the reaction to these examinations. The high school student who submits the best paper on the examination is granted a $500 scholarship to Stanford University. Other participants may be listed for honorable mention.

The examination is prepared and the papers are examined by members of the Department of Mathematics of Stanford University. The problems are intended to be more difficult than those usually treated in high schools in order to test effectively the participants' mathematical ability and understanding.

Annual reports of the contest have appeared in the *American Mathematical Monthly* as follows: Vol. 53, 1946, pp. 406–11; Vol. 54, 1947, p. 430; Vol. 55, 1948, p. 448; Vol. 56, 1949, pp. 496–497. The most complete account appeared in 1946.

The examination given April 2, 1949 was as follows:

1. Prove that no number in the sequence
   11, 111, 1111, 11111, . . .
   is the square of an integer.
2. The three sides of a triangle are of lengths l, m, and n, respectively. The numbers l, m, and n are positive integers,
   $$l \leq m \leq n.$$
   (a) Take $n=9$ and find the number of different triangles of the described kind.
   (b) Take various values of n and find a general law.
3. (a) Prove the following theorem: A point lies inside an equilateral triangle and has the distances x, y, and z from the three sides respectively; h is the altitude of the triangle. Then
   $$x+y+z=h.$$
   (b) State precisely and prove the analogous theorem in solid geometry concerning the distances of an inner point from the four faces of a regular tetrahedron.
   (c) Generalize both theorems so that they should apply to any point in the plane or space, respectively (and not only to points inside the triangle or tetrahedron). Give precise statements and, if you have time, also proofs.

AIDS TO TEACHING

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BOOKLETS

B.28—Milestones in Optical History

Bausch and Lomb Optical Company; Rochester, New York

Pamphlet; 30 pages; free; 6 frames for the pictures, $3.00; 12 pictures framed $6.00.

Description: This brochure contains color reproductions of beautiful paintings and historical sketches of scientists and mathematicians who have contributed to the development of optics. The pictures include Alhazen, Confucius, Galileo, Snell, Huygens, van Leeuwenhoek, Newton, Franklin, Thomas Young, Fraunhofer and Helmholtz. The historical accounts include references to other mathematicians, for example, Euclid and Descartes. Since the development of optics is highly dependent upon mathematical principles, the text contains frequent references to the role of mathematics.

Appraisal: This beautiful brochure is printed on high quality glossy paper that makes possible excellent color reproductions of artistic painting. The pictures of most interest to the mathematics teacher will be those of Galileo, Snell, Huygens, Newton and Fraunhofer. These pictures framed would add much to the atmosphere of a mathematics classroom. The historical sketches are brief, readable and printed attractively. There is very little advertising in the pamphlet. Every mathematics teacher should send for this free brochure even though the amount of mathematical material is limited.

CHARTS

C.12—Plastic-Plated Graph Chart (No. 21)

Modern School Products; P. O. Box 2606; Cleveland 7, Ohio

38×50", $7.75.

Description: The demonstration graph chart is painted in black on a clear white background and mounted on ¼" laminated mounting board. The transparent plastic surface has been applied by heat and pressure and provides a smooth surface upon which the user may draw with wax crayon, tempera, or water color; the surface can then be wiped clean with a soft dry cloth. The graph is ruled in one-inch squares—every fifth line being drawn heavily. Space is left along the left-hand side for recording essential data, titles, or legend. The entire edge of the graph board is masked with heavy black tape for protection.

The chart is extremely attractive and far more professional in appearance than the usual blackboard type. Its durability is dependent upon the care exercised in protecting the plastic surface for when this is pierced drawing is hindered. With reasonable care, however, the surface may be expected to last many years. The visibility of figures drawn upon the graph is excellent.

Appraisal: The chart is sufficiently attractive to provide extrinsic motivation in work with graphs. Although appearance should hardly be accepted as a major criterion of suitability, its stimulative effect upon pupils should not be underestimated.

The chart is equipped with grommets for hanging. It is available in two forms. The one discussed has a plain white reverse side. Model number 22, whose price is also $7.75 has rectilinear coordinate

graphs on both sides. Preference of one model over another is a matter of personal selection. Both charts are well worth the price asked. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

EQUIPMENT

E.24—The Educator Number Fence

The Little Red School House; 38 Main Street; Manasquan, New Jersey

Teachers' Guide Book, $3.00; teachers' (large) demonstration set $1.25; stand for elevating large set, $1.25; individual NUMBER FENCES for each child, $15.00 a dozen.

Description: This individual NUMBER FENCE is a hard wood block 8 by $1\frac{3}{4}$ by $1\frac{1}{4}$ inches. There are ten holes for pegs lengthwise along the top center of the block and a slot cut crosswise midway between each pair of holes. The ten pegs and block are of contrasting colors, and the four celluloid crosspieces which are placed in the slots to separate the pegs into sub-groups are of still another color. There is an oversize model for teacher demonstration.

Appraisal: This device provides a variation in the commonly practiced technique of having pupils manipulate ten or fewer than ten objects in order to develop concepts, relationships, and so on for the numbers 1 to 10. Instead of breaking up and combining subgroups by actually moving them, the cross pieces serve to separate the total group into its component groups. This would probably lead to fewer objects rolling off of desks, but there would be pupils who would not see the subgroups unless there was actually greater spatial separation. The teacher would find the NUMBER FENCE helpful to use along with the bundle of sticks, the groups of disks and whatever other groups of objects she was using, but this device should be just one of several groups of manipulatory objects the child has available.

The base of the device and the pegs are sturdy. The crosspieces can easily fall out and possibly become lost.

The NUMBER FENCE is most suitable for work in the kindergarten and in grades one and two. However, it would also serve as a reteaching device for grades three and above. (Reviewed by Robert L. Burch, Boston University.)

FILMS

F.43—The Language of Graphs

Coronet Films; Coronet Building; Chicago, Illinois

Film; 16 mm, 1 reel; Black and White, $56.25; Color, $112.50; 1948; Collaborator, H. C. Christofferson.

Content: This film shows the staff of a high school newspaper solving the financial and circulation problems of their publication by using bar, line, circle, and equation graphs to picture relationships and make comparisons. Animation is used to show the steps in the drawing of the different types of graphs. The technical vocabulary used is explained and the maining of the graphical results is discussed by the newspaper staff. A comparison of the different types of graphs is used to summarize the film and shots of graphs from foreign countries emphasize the universality of this method of presenting facts.

Appraisal: This film uses a natural school situation to present the methods and uses of several types of graphs in an effective manner. The use of animation in reading a graph is well done. Although the film covers much material in a rapid fashion, the vocabulary is satisfactory and drawings well labeled so that properly prepared students will readily follow the commentary. The shots of graphs from foreign countries is an abrupt break from the content of the balance of the film. The photography and sound are highly satisfactory.

F.44—Percentage

Johnson Hunt Productions; 1133 North Highland Avenue; Hollywood 38, Calif.

Film, 16 mm, sound; 1 reel; black and white, $45.00; color, $85.00. Teaching guide available; 1948.

Content: Through animation, segments of a disk are used to illustrate how a fraction can be changed. The film shows how to change common and decimal fractions to per cent by computation. The meaning of the expression "per cent" is described and illustrated. The painting of a shed, building of a wall and an automobile trip are used to show the use of per cent in life problems. Two methods of solving percentage problems, one using fractions, the other using decimals, are contrasted.

Appraisal: This film, the last of a series of eight, may be useful in developing an understanding of percentage. This is a topic that obviously cannot be treated extensively by a ten minute film. The meaning of percentage and its relation to fractions is shown in an effective manner. However, the applications presented lack appeal for immature pupils. For example, it might have been more effective to apply percentage to games or activities in which most students are vitally interested rather than to the problem of painting a shed or computing the per cent of a highway which is not paved. Although the explanations are similar to those of the classroom, this may be the best way to supplement instruction. The use of animation to change a denominator of 100 to the per cent symbol was effective. The film is recommended for intermediate grades and remedial work in higher grades. The animated photography and commentary are very satisfactory.

INSTRUMENTS

I.16—Contractor's Transit Level (No. 40)

Yoder Instruments; East Palestine, Ohio. $56.50.

Description: The cast brass base of the transit has four leveling screws which are connected to the tripod head. The aluminum horizontal arc is $4\frac{1}{2}''$ in diameter. It is graduated in degrees but a vernier attachment allows readings correct to the nearest six minutes. The vertical arc is likewise calibrated in degrees—eighty degrees in either direction from the vertical. Due to the physical construction of the instrument, angles of elevation and depression may be read up to 45 degrees. The brass telescope is ten inches long and its magnifying power is ten diameters. It is attached to the uprights of the u-shaped standards. Two large adjustable spirit levels facilitate adjusting the instruments. One is secured to the telescope, the other to the standards.

The cast iron tripod head has flanges to which the legs are attached. The 52-inch ash legs are protected at their bases by metal shields.

A steel plumb bob and line complete the set which comes packed in a heavy wooden box. Net weight of the instrument is 10 pounds.

Appraisal: The transit and level are the two most widely used surveying and construction instruments. Their practical nature makes them all the more desirable for teaching numerical trigonometry and for solving mensuration problems. This instrument is somewhat precise for use in schools. It is simple to understand, however, for both protractors are in the open. The vernier is a most desirable feature.

The transit-level may be easily mastered by high school students and represents a valuable addition to the mathematics department. Its expense, however, should make it unavailable except in cases where field work plays an important role and the school budget is liberal. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

MODELS

M.9—Cone, Sphere and Cylinder (74120)

Central Scientific Company; 1700 Irving Park Road; Chicago 13, Illinois.

3″ high, $4.25.

Description: The set consists of a cone

cylinder, and sphere made of sheet copper; each of the figures is three inches high and has a three-inch diameter, thus enabling comparison of the relative volumes of the three solid forms. The models are particularly well-constructed, the cone and cylinder being reinforced by rims at their open ends. Handles are attached to the cylinder and cone so that the teacher will not obstruct the students' view during demonstration. The sphere contains a small hook to facilitate lowering the model carefully into the cylinder whose inner diameter is slightly larger than the outer diameter of the sphere.

Appraisal: This set is intended for use in teaching relations among the volumes of the cylinder, sphere, and cone. The idea is an old one but none the less effective because of its age. By pouring water into the models, it may be proved that the ratio of the volumes of the cone, sphere, and cylinder are 1:2:3.

The demonstration is so vivid that students can hardly fail to understand the relations. Expense will, in most schools, limit the number of purchased sets to one demonstration set; similar sets may, however, be prepared by students. The sphere may be made of varnished wood whereas the cylinder and cone may be made in the school sheet-metal shops in quantities sufficient to allow teaching the concepts involved by actual experimentation.

The formula for determining the volume of a cylinder may be taught in the traditional manner and the other two formulas developed experimentally from it.

These models, whether purchased or student-prepared, should certainly appear in the classrooms and laboratories where the volumes of cones, spheres and cylinders are studied. (Reviewed by Bernard Singer; Hyannis, Massachusetts.)

PICTURES

P.8—Diagrams in Three Dimensions for Solid Geometry

The Orthovis Company; 1328 South Wabash; Chicago, Illinois.

8×9″, $.80.

Description: This set consists of fifteen geometric stereograms which illustrate theorems and definitions studied in solid geometry courses. The stereograms are printed in red and blue on white cardboard and two orthoscopes are supplied for viewing. Five cards are devoted to three-dimensional representations of Planes and Lines in Space, four to Polyhedrons, and six to the Sphere. Each card contains a statement of the theorem or definition illustrated.

The anaglyphs are cleverly drawn but the red and blue inks do not match the orthoscope lenses, undesired lines are thus somewhat visible during viewing. The three-dimensional illusion is there, nevertheless.

Each card is numbered for easy reference. Several inches of blank space at the base of each card enable the teacher to make a permanent, convenient record of significant problems or information which apply to the diagrams.

Appraisal: The anaglyphs are useful for helping pupils to visualize three-dimensional figures so that they may devote their attention more economically to proving the theorems. They may be used to rectify misinterpretations derived from viewing two-dimensional diagrams or merely to verify their spatial interpretation.

Anaglyphs present a feasible method of representing three dimensions but the strain produced by color rivalry between the blue and red makes this device inferior to the stereograph.

The greatest advantage of this set is derived from the interest it promotes; students enjoy supplementing their attempts at space perception with the novelty of an anaglyph.

These aids are for individual study and should be placed where students will have the opportunity to study them when needed. The price is commensurate with the educational value. (Reviewed by Bernard Singer; Hyannis, Massachusetts.)

SOURCES OF MATERIAL FOR LABORATORY WORK

SL.12—Domestic Measures and Measuring Devices

Radio Script and Transcription Exchange; United States Office of Education; Washington, D. C.

Transcription: 16 inches in diameter; 33⅓ rpm; plays inside out; available for cost of return postage to Washington.

Description: This transcription is a recording of one of the "Adventures in Research" series produced for the radio by Westinghouse Electric. By using the interview technique, an announcer leads a member of the Westinghouse staff into a description of measures and devices to measure. The following topics are covered: weight, mass vs. weight, length (including a comparison of the English and Metric systems), humidity, volume, energy, some ways to measure the quantities described above, the need to beware of false measuring devices, the measure of time (including the sundial, waterclock, hour-glass, wooden and electric clocks), and a final discussion of various electrical units.

Appraisal: The idea behind the transcription is excellent and may serve as inspiration for teachers and pupils to write and record similar descriptions of mathematical topics, but the usefulness of this particular transcription in the classroom is very slight. The interview method of extracting information is a poor way to conceal a lecture; a dramatization of some important moments in the development or use of measuring devices would have been more interesting. The organization of the material is confusing, as one may see by reading the list of topics above. The discussion of weight, mass, humidity, electrical clocks and electrical units were suited to physics students with some little background; on the other hand, the descriptions of length, volume, energy, sundials, waterclocks, and hourglasses could be used as low as the fifth grade. Similar material, carefully designed for an appropriate level, would be most welcome in mathematics classes.

SL.13—Measurement Exercise (MO 208)

The Mathaids Company; 204 Forman Avenue; Syracuse, New York.

$1.50 per set.

Description: The set consists of forty rectangular cards ranging in size up to 9¾ inches in length and 5½ inches in width; no two cards have the same dimensions; each is numbered for recognition. The cardboard used is very thin but durable nevertheless.

Appraisal: The cards have been designed for the use of students while studying perimeter and area of rectangles. They should be distributed among the students and as each completes work on one rectangle, he passes it to the next student, until each has finished a predetermined number. The identification numbers on the cards then enable the teacher to measure the progress of the class.

The idea of measuring cards is more concrete and meaningful to most students than the solution of traditional textbook problems; this type of work should be supplemented by measurement of other convenient rectangles within the room.

$1.50 is an unreasonable price for a set of forty cards which may be easily made by any teacher in a very short time. (Reviewed by Bernard Singer; Hyannis, Massachusetts.)

SL.14—Metric Measure Set (41–45)

Cambosco Scientific Company; 37 Antwerp Street; Brighton Station, Massachusetts.

10 c.c.–1,000 c.c. inclusive, $8.25.

Description: The Metric Measure Set consists of seven cylindrical metal cups whose volumes range from ten cubic centimeters to one liter. The cups are formed of cut brass tubing one of whose ends is sealed with a brass disk soldered into

place. Each cup has its capacity permanently engraved upon its side. The containers are very sturdy and should last indefinitely.

Appraisal: The containers are intended for use in teaching relationships among metric volumes. These relationships are probably best taught by allowing the students to "discover" them through experimental methods. The containers may then be displayed for several days to allow students to become more familiar with them. The price of the set is obviously high for most schools. There is little justification in purchasing it unless it is primarily for the science department. (Reviewed by Bernard Singer; Hyannis, Massachusetts.)

SL.15—One Inch Cube Set (B 205)
The Mathaids Company; 204 Forman Avenue; Syracuse, New York.
(2×3×6″), $1.00.

Description: Thirty-six one-inch black walnut cubes constitute the set. The blocks are accurately cut and the hardwood is extremely durable.

Appraisal: The wooden blocks are useful for visualizing fractional concepts and for indicating the meaning behind the fundamental operations as they apply to common and decimal fractions. The set is useful, also, for deriving intuitively, formulas for the volume of a rectangular solid and cube; cubes and rectangular prisms of differing dimensions may be formed for determining ratios of their volumes.

The use of concrete objects for the purposes mentioned above is more meaningful to most students than is the solution of abstract problems; the blocks in question have two major disadvantages—they are too small and too difficult to handle for effective demonstration. (Reviewed by Bernard Singer; Hyannis, Massachusetts.)

CORRECTION

The information given for P. 4 in the Aids to Teaching column in the November 1949 issue should have the address corrected to read, Stanley Bowmar, 513 West 166th Street, New York 32. New York.

How will they look to YOU a few years from now?

Your wife's eyes: What will you read in hers when she asks whether you can afford that modest cottage that's for sale?

Your boy's eyes: What will you see in his eyes the day he asks whether you can afford to send him to college?

Your own eyes: What will the mirror tell you about them when it's time to retire, and take things easier?

There's no better time than right now to sit back and think what *you* will see in your family's eyes a few years from now.

Whether they glow with happiness or turn aside with disappointment depends, to a very large extent, upon what you do *now*.

So plan *now* for that home you plan to buy eventually . . . set aside money *now* for his college education . . . plan *now* for the day you can retire.

Decide now to put part of your salary week after week, year after year in U. S. Savings Bonds, so that you will have the money for the *important* things you and your family want.

Insure your future by signing up on the Payroll Savings Plan where you work, or the Bond-A-Month Plan where you have a checking account.

Chances are you won't miss the money now, but you certainly will a few short years from now if you haven't got it!!

P. S. Remember, too, that every $3 you invest now in U. S. Savings Bonds returns $4 to you in just ten short years.

Automatic saving is sure saving — U.S. SAVINGS BONDS

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